RUME XVII CONFERENCE SCHEDULE

1:00 - 1:15 pm
Grand Ballroom
Salons 2-4

OPENING SESSION

1:25 - 1:55 pm

SESSION 1 – CONTRIBUTED REPORTS

City Center A

Abstract Algebra and Secondary School Mathematics: Identifying Mathematical Connections in Textbooks

Ashley Suominen

Undergraduate students learning abstract algebra for the first time often struggle to relate the new abstract concepts to previously learned concepts found in secondary school mathematics. The goal of this research was to examine nine introductory abstract algebra textbooks to elaborate on the mathematical connections that authors explicitly mentioned in the text between abstract algebra and secondary school mathematics concepts. For the purpose of this study, mathematical connections are viewed as a characteristic of mathematics, where a mathematical connection is defined as a relationship between two concepts. In this report of my textbook analysis, I discuss two specific categories of connections: comparison through common features and hierarchical or inclusion. This study forms the basis of a further investigation into mathematicians' and mathematics educators' perspectives on these mathematical connections.

Paper

Grand Ballroom Salon 5

It's about time: How instructors and students experience time constraints in Calculus 1

Jessica Ellis, Estrella Johnson and Chris Rasmussen

The goal of this research is to better understand the relationship between how quickly or deeply Calculus material is covered and how this is related to students' instructional experience and their persistence in a STEM major. To do so, we analyze data coming from a large national survey of Calculus I programs. Specifically, we first compare students' views of pacing to their instructor's views, resulting in four classifications of students. We then investigate various characteristics of these students, including: their institution type, their instructor type, their reported instruction, their mathematics beliefs, and their Calculus II intentions. Our findings suggest two important ways that pressure to cover material impacts Calculus I students and their instructors. First, when instructors feel pressure to cover material, student-centered teaching practices are often dropped. Second, feeling rushed to cover difficult material is a factor in losing STEM intending students.

Paper

Marquis Ballroom B

Opportunity to learn the concept of group in a first class meeting on abstract algebra

Tim Fukawa-Connelly

This paper is a case study of the teaching of an undergraduate abstract algebra course; in particular, it examines the opportunity that the students had during the first class meeting to learn about the concept of a mathematical group and how much intellectual responsibility for the definition they were given. The paper offers a description of the classroom teaching and discussion an inquiry-oriented abstract algebra course. It focuses on the mathematical tasks that the professor used to introduce the concept of a group, the student solutions and classroom presentation of those tasks, and the professor's activities. It analyzes the class in terms of providing students the opportunity to learn the concept of a mathematical group and about the mathematical process of defining. It also analyzes the professor's activities in terms of devolving intellectual responsibility for writing the definition of a group. Finally, the analysis suggests ways that the professor's actions failed to devolve mathematical authority and limited opportunities to learn.

Paper

2:05 - 2:35 pm

City Center A

Generalization in undergraduate mathematics education

Allison Dorko, Eric Weber and Steven Jones

Generalization is a critical component of mathematical thought and thus a goal of instruction. However, most generalization research has focused on pattern generalization and generalization in algebra, not undergraduate mathematics. It is not known whether the generalization frameworks derived from such work adequately describe generalization in undergraduate mathematics, where the sorts of generalizations students must make are not regarding patterns, but rather the meanings of concepts and ideas. In this paper, we propose elaborations on existing generalization frameworks in order to take into account issues in learning advanced mathematics. We use how students generalize their notion of integration from single to multiple integrals as an illustrative case study.

Paper

City Center B

A preliminary categorization of what mathematics undergraduate students include on exam "crib sheets"

Antony Edwards and Birgit Loch

Many undergraduate examinations permit students to use a limited quantity of previously prepared notes: so-called "crib sheets", or "cheat-sheets". The majority of evidence from the literature suggests that students sitting such exams feel less anxious, and that they perform to a higher standard, although such results are idiosyncratic to the discipline and course, and few are set in the context of undergraduate mathematics. Less is known about what content students choose to include on their sheets, and how they interact with this material. This preliminary research report presents the first results from a three-year project investigating students' use of crib sheets in undergraduate mathematics exams. It explores the content and layout of crib sheets used by students for an end-of-semester calculus exam.

Paper

Marquis Ballroom A

The textbook, the teacher, and the derivative: Examining community college instructors' use of their textbook when teaching about derivatives in a first semester calculus class

Linda Leckrone

Initial qualitative work with five community college calculus instructors indicated that these teachers not only modified their textbook when introducing the concept of the derivative, but also formally and informally evaluated their text. In addition, during classroom observations and interviews, these teachers did not distinguish clearly between the idea of the derivative as an object or an operator. This preliminary report details these findings and proposes questions for further research.

[Paper](#)

Grand Ballroom Salon 5

Discourse in mathematics pedagogical content knowledge

Shandy Hauk, Allison Toney, Reshmi Nair, Nissa Yestness and Melissa Troudt

What is happening for in-service teachers at the classroom intersection of mathematics, culture(s), teaching, and learning? How can knowing the answer to that question inform teacher preparation, induction, and development? In ongoing efforts to model and measure the intercultural and relational aspects of pedagogical content knowledge, we present a model and data analyses. The focus is teacher learning and intercultural orientation development. Data are pre- and post-program written tests, surveys, and classroom observations among four cohorts (70 in-service teachers) enrolled in a two-year master's program. The focus at the conference is harvesting the intellectual power of the audience to consider questions about the connections – qualitative, quantitative, and otherwise – among core constructs in pedagogical content knowledge, the thinking that teachers do in connecting them, and how knowing about this can inform teacher education and professional development.

[Paper](#)

Marquis Ballroom B

Components of a formal understanding of limit

Stephen Strand

This presentation reports on research investigating what mathematical constructs and ways of understanding constitute a formal understanding of limit. This work builds primarily on the genetic decomposition of Swinyard and Larsen (2012), which itself was modified from Cottrill, et al. (1996). Ten undergraduate students were interviewed in a semi-structured, clinical setting. While analysis is in the preliminary stages, the data suggest that the types of tasks given have a significant influence on whether or not formal understanding is demonstrated.

[Paper](#)

*Marquis
Ballroom C*

An analysis of sociomathematical norms of proof schemes

Brian Katz, Rebecca Post, Milos Savic and John Paul Cook

We report on a case study aimed at researching the social interactions of a classroom focusing on the certainty of mathematical claims and justifications. Blending Harel and Sowder's (1998) concept of "proof schemes" with Yackel and Cobb's (1996) "sociomathematical norms," we aim to expand on Fukawa-Connelly's (2012) research on sociomathematical norms of proof presentations. Preliminary analysis of classroom interaction and student interview transcripts from a proof-based, axiomatic geometry course suggests the presence of sociomathematical norms related to argumentation that lie outside of proof validation that facilitate renegotiating proof schemes.

Paper

2:35 – 3:05 pm   COFFEE BREAK

3:05 – 3:35 pm

*Grand Ballroom
Salon 5*

Mathematicians' uses of examples when developing conjectures

Elise Lockwood, Alison G. Lynch, Amy B. Ellis and Eric Knuth

This paper explores the role examples play as mathematicians formulate conjectures. Although previous research has examined example-related activity during the act of proving, less is known about how examples arise during the formulation of conjectures. We interviewed thirteen mathematicians as they explored tasks requiring the development of conjectures. During the interviews, mathematicians productively used examples as they formulated conjectures, particularly by creating systematic lists of examples that they examined for patterns, an activity that we call "Data Collection." The results suggest further research and pedagogical implications for explicitly targeting examples in conjecturing, and the study contributes to a body of literature that points to the benefits of exploring, identifying, and leveraging examples in proof-related activity.

Paper

*Marquis
Ballroom A*

Roles of proof in an undergraduate inquiry-based transition to proof course

Sarah Bleiler and Jeffrey Pair

De Villiers (1990) suggested five roles of proof important in the professional mathematics community that may also serve to meaningfully engage students in learning proof: verification, explanation, systematization, discovery, and communication. We investigate written reflections on an end-of-semester assignment from undergraduates in an inquiry-based transition to proof course, where students reflected on instances during the semester when they engaged in the five roles of proof. We present (a) student rankings of role engagement, (b) the types of activities students recalled as influential to their engagement in the roles of proof, and (c) how students perceived they engaged in the five roles. Students in this course reflected on activities distinctive of the inquiry-based environment (such as discussing, presenting, conjecturing, and critiquing) as influential to their engagement in the roles of proof. We provide student quotations highlighting these activities and offer implications for both research and practice.

Paper

*Marquis
Ballroom B*

An investigation of beginning mathematics graduate teaching assistants' teaching philosophies

Kedar Nepal

This qualitative study is an investigation of the teaching philosophies of beginning mathematics graduate teaching assistants (MGTAs). The study considered the cases of two domestic and two international MGTAs. Three teaching philosophy statements from each of the participants were collected at three different stages of a semester-long teaching assistant preparation program (pre-service phase). Three one-on-one interviews were conducted with each participant in the following four semesters (in-service phase) after the conclusion of the pre-service preparation program course. These audio-recorded interviews were transcribed and analyzed using the constant comparative method. Beginning teaching philosophies of these participants and how their philosophies changed over time, during both the pre-service and the in-service phases, will be briefly discussed. The factors that influenced their teaching philosophies in both phases will also be discussed.

[Paper](#)

*Marquis
Ballroom C*

Leveraging historical number systems to understand number and operation in base 10

Eva Thanheiser and Andrew Riffel

Historical number systems are used to build preservice teachers (PTs) understanding of base 10. Variation theory states that you cannot know something if all you know is that one thing. Thus to understand numbers and operations in our base 10 system, PTs need to experience number and operations within and beyond our base 10 system. In this study I examined whether and how historical numbers systems can be used to allow PTs to build a conceptual understanding of numbers and operation in base ten system. Thirty-six PTs concepts of number were identified before and after they work through a series of tasks situated in various number systems and compared and contrasted across the systems. Almost all PTs developed a more sophisticated conception of number though these experiences.

[Paper](#)

3:45 – 4:15 pm

Marquis Ballroom A

Pedagogical challenges of communicating mathematics with students: Living in the formal world of mathematical thinking

Sepideh Stewart, Ralf Schmidt, John Paul Cook and Ameya Pitale

In this paper we examine an Abstract Algebra professor and one of his students' thought processes simultaneously as the class was moving toward the proof of the Fundamental Theorem of Galois Theory. We employed Tall's theory of three worlds of mathematical thinking to trace which route (embodied, symbolic, formal) the mathematician was choosing to take his students to the formal world. We will discuss the pedagogical challenges of proving an elegant theory as the events unfolded.

Paper

Grand Ballroom Salon 5

The Structure, content, and feedback of Calculus I homework at doctoral degree granting institutions and the role of homework in students' mathematical success

Jessica Ellis, Kady Hanson, Gina Nunez and Chris Rasmussen

In this study we investigate the relationship between the nature of Calculus I homework and student success. We examine these connections at five PhD granting institutions that were identified in a large US national study as having relatively successful Calculus I programs (compared to similar institutions) and we draw on both qualitative and quantitative from this study. Mixed method analyses point to a clear relationship between homework systems with varied structure, high feedback, and varied content emphases at more successful Calculus I programs where students persist onto Calculus II at higher rates and where students maintain more positive dispositions towards mathematics.

Paper

Marquis Ballroom B

Teachers' meanings for the substitution principle

Stacy Musgrave, Neil Hatfield and Patrick Thompson

Structure sense is foundational to mathematical thinking. This report explores high school math teachers' meanings for the substitution principle, a sub-category of structure sense that research previously identified as sources of difficulty for students. A focus on meanings reflects our belief that teachers' meanings directly impact the mathematical meanings students develop. We suggest ways of thinking that could lead to various response types as a resource for teacher educators to design professional development targeting improved structure sense for teachers.

Paper

Marquis
Ballroom C

A model of the structure of proof construction

Tetsuya Yamamoto

This paper offers a framework for modeling "the structure of proof construction," which is a comprehensive view that can encompass the aspects, factors, patterns, and features involved in cognitive processes in proof construction across mathematical subjects. The framework was created for the purpose of analyzing students' difficulties with proving. The model was built by applying the think-aloud method to the researcher's cognitive actions while solving dozens of proofs from multiple undergraduate advanced mathematical subjects. The resultant model can serve as a tool to understand the sources of students' difficulties and as methodological and metacognitive knowledge to help students with proof construction. The findings may contribute to a body of knowledge of pedagogical approaches for teaching proofs and to the development of a theoretical framework for proof construction.

Paper

4:20 – 4:50 pm

Grand Ballroom
Salon 5

Connecting research on students' common misconceptions about tangent lines to instructors' choice of graphical examples in a first semester calculus course

Brittany Vincent and Vicki Sealey

Common misconceptions that students have about tangent lines are well documented in the research literature. This study seeks to understand the efforts that instructors make to address these common misconceptions in their classroom instruction. Specifically, we looked at video data from classroom sessions of five instructors when they covered the graphical representation of derivative. Language and gestures instructors used as well as the graphical examples they provided to the students were analyzed.

Paper

City Center A

Undergraduate students' experiences in a remedial mathematics classroom

Durrell Jones and Beth Herbel-Eisenmann

Understanding students' perspectives about their mathematics classroom experience is important for supporting students' learning. As part of a larger study, 15 students enrolled in enrichment sections of a developmental mathematics course were interviewed to explore students' experiences and dispositions. We highlight areas that students described as helpful or not and the ways in which students' framed themselves as having a "productive disposition" (National Research Council (NRC), 2001). Preliminary findings suggest students found activities most helpful when they thought that the activity might help them be successful on other activities and that they described themselves as having many characteristics of a productive disposition. A more detailed analysis of these descriptions, however, uncovered narrow interpretations of words like "understanding" and "making sense" in mathematics.

Paper

City Center B	**Differentiating instances of knowledge of content and students (KCT): Responding to student conjectures**

Kristin Noblet

This study investigates the nature of preservice elementary teachers' knowledge of content and teaching (KCT) through responses to hypothetical student scenarios. Participants demonstrated two types of KCT: (1) specific KCT, which drew from the specific mathematics of the student scenario and most resembles the construct described by Ball, Thames, & Phelps (2008); and (2) general KCT, which presented itself as "canned" mathematical pedagogy. Examples and influences upon each type of KCT are explored to promote discussion on the differentiation of KCT and for further refinement of the concept of general KCT.

[Paper]

Marquis Ballroom A	**Business faculty perceptions of the calculus content needed for business courses**

Melissa Mills

This paper uses an online survey instrument to explore what Calculus topics business faculty view as relevant to, and necessary for, various business specializations. This is part of a larger study that aims to design a Calculus course that better addresses the mathematical needs of both business students and business faculty.

[Paper]

Marquis Ballroom B	**Knowledge for teaching: Horizons and mathematical structures**

Nicholas Wasserman and Ami Mamolo

In this study we use the lens of knowledge at the mathematical horizon to shed light on the underlying structural components of school mathematics content. We attend specifically to algebraic structures, identifying ways in which awareness of such structure may be transformative to mathematical knowledge for teaching. In particular, we analyze curricular content from elementary, middle, and secondary school mathematics with respect to the inherent structures that encompass and connect that material, as well as with specific attention to the opportunities afforded to teachers by their horizon.

[Paper]

Marquis
Ballroom C

Undergraduate students reading and using mathematical definitions: Generating examples, constructing proofs, and responding to true/false statements

Valeria Aguirre Holguin

University students often encounter difficulties making correct use of definitions, partly because they are not familiar with the difference between dictionary definitions and mathematical definitions (Edwards & Ward, 2004), yet in order to succeed in their upper-level mathematics courses, they must often construct original (to them) mathematical proofs, which intrinsically require the correct use of definitions. Only a little research has been conducted to discover how university students handle definitions new to them (e.g., Dahlberg & Housman, 1997). Our research question is: How do university students use definitions, in particular to evaluate and justify examples and non-examples, in proving, and to evaluate and justify true/false statements? Data were collected through individual task-based interviews with volunteer students from a transition-to-proof course. There were five definitions but each student was asked to consider only one of the five. Content analysis and grounded theory are being used for analysis. Preliminary results are presented.

Paper

5:00 – 5:30 pm

Grand Ballroom Salon 5

How do mathematics majors translate informal arguments into formal proofs

Dov Zazkis, Keith Weber and Juan Pablo Mejia-Ramos

In this paper we examine a commonly suggested proof construction strategy from the mathematics education literature—that students first produce an informal argument and then use this as a basis for constructing a formal proof. The work of students who produce such informal arguments during proving activities was analyzed to distill three activities that contribute to students' successful translation of informal arguments into formal proofs. These are elaboration, syntactification, and rewarranting. We analyze how attempting to engage in these activities relates to success with proof construction. Additionally, we discuss how each individual activity contributes to the translation of an informal argument into a formal proof.

Paper

Marquis Ballroom C

Developing a creativity-in-progress rubric on proving

Milos Savic, Gulden Karakok, Gail Tang and Houssein El Turkey

There is a considerable amount of mathematics education literature on creativity (e.g., Torrance, 1966; Balka, 1974; Silver, 1997), yet there is little discussion of mathematical creativity in undergraduate mathematics education. Specifically, to our knowledge, there is no literature on mathematical creativity in the proving process. We attempt to contribute to the literature by developing a framework in which to discuss mathematical creativity in proving, Creativity-in-Progress Rubric (CPR) on proving. Through previous rubrics on creativity (Leikin, 2009; Rhodes, 2010) and interviews with both mathematicians and undergraduate students, we claim that there are three aspects of creativity in proving: Making Connections, Taking Risks, and Creating Ideas. We demonstrate how to use the rubric with an example of a student's proving process. Finally, we give future research considerations of using the rubric in proof-based classrooms.

Paper

*Marquis
Ballroom B*

Application of multiple integrals: From a physical to a virtual model

Ivanete Siple and Elisandra Figueiredo

This paper describes the evolution of a teaching technique for calculating volume using multiple integrals - from a physical to virtual model - in a Differential and Integral Calculus (CDI) course involving students in math and engineering programs at a public university in Brazil. This transition was made possible by the use of e-learning tools on the Moodle platform, which is used to support classroom activities. As a result of this technique, we saw the deepening of the concepts of parameterization taught in analytic geometry that were necessary to build the model using graphics software (virtual) as well as a change in the dynamic of sharing the technique among teachers and students through discussion forums in a continuous process.

Paper

*Marquis
Ballroom A*

Pre-Service Teachers' Inverse Function Meanings

Teo Paoletti, Irma E. Stevens, Natalie L. F. Hobson, Kevin R. Laforest and Kevin C. Moore

The concept of inverse function is included in The Common Core State Standards of Mathematics (National Governors Association Center for Best Practices, 2010) under the function content strand. Thus, it is important for pre-service teachers to develop productive meanings for inverse function. However, researchers have indicated that pre-service teachers (along with in-service teachers and college students) typically do not develop such meanings. In order to explore pre-service teachers' inverse function meanings further, we conducted a series of clinical interviews with 25 pre-service teachers. In this proposal, we include a summary of previous research concerning individuals' inverse function meanings as well as a description of the methodology and theoretical framework we used when making sense of the students' activity. We present select data highlighting both major trends in the students' activity as well as instances that perturbed the students. We conclude with implications from our findings and areas for future research.

Paper

POSTER SESSION & RECEPTION

The Efficacy of projects and discussions in increasing quantitative literacy outcomes in an online college algebra course

Luke Tunstall

This research stems from efforts to infuse quantitative literacy (QL) in an online version of college algebra. College algebra fulfills Appalachian's QL requirement, and is a terminal course for most who take it. In light of the course's traditional content and teaching methods, students often leave with little gained in QL. An online platform provides a unique means of engaging students in quantitative discussions and research, yet little research exists on online courses in the context of QL. The researcher's course includes weekly news discussions as well as "messy" projects requiring data analysis. Students in online and face-to-face sections of the course took the QLRA (developed by the NNN) during the first and final weeks of the fall 2014 semester. There were significant statistical gains in the online students' QLRA performance and mathematical affect but none for the face-to-face students. Implications of this include that project-based learning in an online environment is a promising strategy for fostering QL in terminal math courses.

Paper

Students' reasoning about marginal change in an economic context

Thembinkosi Mkhatshwa and Helen Doerr

This study reports on how ten undergraduate students enrolled in business calculus reasoned about marginal change (marginal cost, marginal revenue, and marginal profit) while engaged in a task-based interview followed by a semi-structured interview. The study had two major findings: (1) students had difficulty distinguishing between marginal cost and approximate marginal cost and (2) students conflated marginal cost and marginal revenue with total cost and total revenue respectively. For future research, we might consider investigating how students' understanding of marginal change impacts their ability to solve optimization problems situated in the context of cost, revenue, and profit.

Paper

Challenges and resources of learning mathematics in English for a 'mathematically intelligent' student weak English background

Balarabe Yushau

Lack of Proficiency in English is one of the major obstacles in students learning of mathematics in English medium universities in Saudi Arabia. Despite, you still come across some students doing exceptionally good in exams that are conducted in English – a language that the students are not proficient at. In this paper, we intend to share the students' challenges of learning mathematics in English, and the resources that help them to overcome the problems.

Paper

Code-switching and mathematics assessment: Some anecdotal evidence of persistence of first language

Balarabe Yushau

One of the most difficult things for a monolingual teacher to decide is if errors by a student who is acquiring English reflect a lack of mathematical understanding or some problems with English (Secada & Cruz, 2000). Here, we buttress this point with some anecdotal evidence from mathematics works of Preparatory Year University student. It shows the dominance of students' first language (Arabic) over the language of teaching and learning (English) while students are doing mathematics. Furthermore, we shall show how that conflict can affect students understanding and performance in mathematics.

Paper

Students' conceptions of rational functions

Nicholas Fortune and Derek Williams

Mathematics education research on dynamic technologies incorporated into learning environments indicates that they possess the ability to enrich students' mathematical conceptual understanding. This study explores how three community college students conceptualize rational functions by classifying their mathematical thinking according to the APOS theory. This study highlighted students' conceptualization of rational functions as actions (required sequences of observable external steps), processes (sequences with no need to externalize), and objects (thinking about rational functions as a single object or entity).

Paper

Using journals to support student learning: The case of an elementary number theory course

Christina Starkey, Hiroko Warshauer and Max Warshauer

The 17 undergraduates in this course submitted weekly journals online to their instructor and reflected on their mathematical learning. The instructor provided comments to each of the students' journal submissions that informed him of each student's successes, challenges, issues, and questions. We analyzed the journals and share our preliminary findings on what the journal writing revealed about students' learning and how their mathematical understanding developed over a semester. We include results of the pre-post survey of student attitudes toward mathematics along with interviews of 5 of the students that give additional insight into their experiences in the course. Future work will examine the uses of journals in other courses, and different ways journals support student learning.

Paper

Developing abstract knowledge in advanced mathematics: Continuous functions and the transition to topology

Daniel Cheshire

Despite intuitive foundations, the nature of the transition to abstract topology often results in students' reliance on dissociated collections of definitions and theorems, without any integrated cognitive structure. In recent decades, there have been numerous analyses of proof, symbols, and the encapsulation of processes as factors in student comprehension, as well as content-specific studies examining which mental constructions support the development of coherent schemata for particular topics. I will expand this research by categorizing students' understanding in the domain of topology. In a two-semester, mixed-methods study, I will analyze the components involved in the development of an axiomatic schema for continuous functions in topological contexts. I will compare this model with actual student constructions in an introductory topology course, collected through task-based interviews and a path-analysis on the coded data. The goal is to confirm the theoretical model, or to provide support for altering the model to increase its validity.

Paper

Prospective secondary mathematics teachers' (PSMTs') understanding of abstract mathematical structures

Younhee Lee

Kilpatrick (1987) noted, "successful teaching, like successful communication, depends on having a good model of the other" (p. 17). Successful teaching of collegiate mathematics for PSMTs will necessitate understanding of what knowledge PSMTs may bring with them to the learning of collegiate mathematics and through what processes PSMTs construct their knowledge. Thus, in-depth analysis of PSMTs' mathematics is of significant importance to our field for providing PSMTs with meaningful learning experiences in mathematics content courses. In this study, I intend to investigate how PSMTs construct their own knowledge of abstract mathematical notions (e.g., polynomial rings, irreducible polynomial, factorization, minimal polynomials) and, second, the difficulties that they encounter while constructing their knowledge of abstract mathematical notions.

[Paper](#)

When mathematicians grade students' proofs, why don't the scores agree?

Robert C. Moore

This poster reports on a study of practices that mathematics professors use to grade proofs. In an initial study, four mathematicians evaluated and scored six proofs of elementary theorems written by students in a discrete mathematics or geometry course. The results indicated that, while the professors generally agreed in their overall evaluations of the proofs, the scores varied substantially. A follow-up study delved more deeply into the reasons for the spread in the scores. This poster presents four reasons why the professors did not always agree in their scoring of the proofs: (a) performance errors, (b) disposition toward grading, (c) judgments about the student's level of understanding and the seriousness of errors, and (d) contextual factors.

[Paper](#)

Formal logic and the production and validation of proof by university level students

Sarah Mathieu-Soucy

The goal of our study is to discuss how knowledge of formal logic changes the way students produce and validate proofs in the context of undergraduate mathematics. Also, we seek to understand how those differences revolve around intuitions, contextual reasoning rules and syntactic and semantic productions. With that in mind, we asked 8 students with varied levels of knowledge and different academic background in formal logic to produce and validate proofs during interviews and we analyzed their work. Our analysis suggests that a course in logic changes the way students do mathematical work more significantly than actual knowledge of formal logic. The poster presents the context and main results.

Paper

Unconventional use of mathematical language in undergraduate students' proof writing

Kristen Lew and Juan Pablo Mejia-Ramos

There is a dearth of research on students' use of mathematical language, particularly when writing proofs at the undergraduate level. In this exploratory study, we analyze written student exams (N=149) from an introductory proof course in order to identify different ways in which students' use of written mathematical language differs from mathematicians' writing in formal mathematical settings.

Paper

Embodied world thinking: The calculus laboratory

Sepideh Stewart

The aim of the study was to provide an opportunity for students to experience calculus first-hand and discover many fascinating aspects of the subject using the free software, Geogebra, in weekly lab sessions. The question to consider is: Did visualizing complex and fancy equations such as $y^2 (y^2 - 4)=x^2 (x^2 -5)$ (devil's curve); looking for limits of functions and estimating local maximum and minimum values graphically and other similar examples help a group of 50 students (non-mathematics majors) to relate to what they were studying and connect them closer to calculus? The result of an online survey revealed students' reactions to the lab sessions and their possible effects on learning.

Paper

Understanding participants' experiences in a flipped large lecture calculus course

Erin Glover

There is a growing body of literature that suggests students do better in classes that implement active-learning strategies (e.g., group work, problem solving). Flipping classroom instruction is becoming a popular innovation to support active learning in the classroom. The study reports on one large lecture calculus course where instruction was flipped in order to employ active-learning strategies. This poster will highlight the experiences of the large lecture instructor, recitation leaders, and student focus groups.

[Paper](#)

Learning in one classroom: Developmental mathematics students and prospective mathematics teachers

Kenneth Bradfield, Raven McCrory, Aditya Viswanathan and Kristen Bieda

Developmental mathematics courses in the United States continue to lack in curriculum and instructional practices that promote students' mathematical proficiency. The instructional practices that researchers argue can promote students' mathematical proficiency in K-12 classrooms can apply to undergraduate classrooms as well. This poster will discuss an NSF-funded research project that facilitates students' mathematical development in a non-credit-bearing developmental mathematics course, in concert with providing prospective mathematics teachers an opportunity to learn to teach for mathematical proficiency. The project team collected quantitative data that compared the intervention students to their peers before and after participation in the course. Results indicated that developmental mathematics students who participated in our intervention started behind, caught up, and experienced more success than their peers in their subsequent mathematics course.

[Paper](#)

An examination of college students' reasoning about trigonometric functions with multiple representations

Soo Yeon Shin

The purpose of this study is to examine how individual college students reason through tasks using trigonometric functions and translate among different types of representations of trigonometric functions across various mathematical tasks.

[Paper](#)

An RME-based instructional sequence for change of basis and eigentheory

Megan Wawro, Michelle Zandieh, Chris Rasmussen and Christine Larson

Linear algebra is widely viewed as pivotal yet difficult for university students, and hence innovative instructional materials are essential. The goals of this NSF-funded research project include producing: (a) student materials composed of challenging and coherent task sequences that facilitate an inquiry-oriented approach to the teaching and learning of linear algebra; and (b) instructional support materials for implementing the student materials. This poster will highlight the third unit of the IOLA (Inquiry Oriented Linear Algebra) materials that focus on a research-based approach to introducing eigentheory, change of basis, and diagonalization.

[Paper](#)

Students' visual attention while answering graphically-based Fundamental Theorem of Calculus questions

Rabindra Bajracharya, John Thompson and Jennifer Docktor

As part of work on student understanding of the Fundamental Theorem of Calculus (FTC) and definite integrals, we incorporated a technique known as eye tracking to investigate how students attribute their visual attention while answering graphically-based questions. The direction and duration of eye gaze of 17 students was recorded in real time. We analyzed the total proportion of time spent on various question domains (lexicons, equations & symbols, graphs, and question options) as well as on various relevant and irrelevant features of the graphs. We found that the students who responded correctly spent more time on relevant graphical features, whereas those responded incorrectly spent more time on irrelevant graphical features. We also found that student visual attribution depends on types of representations and notations provided in the questions. Most of the eye-tracking results corroborate previously reported written and interview results on student application of the FTC across the mathematics-physics interface.

[Paper](#)

Domain, Co-domain and causation: A study of Britney's conception of function

Nathan Phillips

Function has been shown to be an equally important, but difficult concept for students to master (Carlson, Oehrtman & Engelke, 2010; Dubinsky & Harel, 1992). Through a clinical interview with a preservice mathematics teacher, I characterize the ways in which her function definition is able to account for novel relationships between quantities. Utilizing APOS theory, I find that though she is able to exhibit a process view of function, the student struggles to reconcile her definition of function with her intuitions about domain/co-domain and causation. The research is part of a larger study examining the ways in which preservice teachers define function affect their ability to accommodate novel contexts and representations.

Paper

An intended meaning for the argument of a function

Ashley Duncan

This poster describes an intended meaning for the argument of a function when reasoning about a function covariationally. An instructional investigation was designed to promote this meaning with students in a college-level precalculus course using an instructional task and a didactic object relating to modeling the relationship between the time elapsed since Tommy threw a rock off of a bridge in seconds and the height above a lake in feet of the rock. Students were formally introduced to the term argument after the task and asked to evaluate and explain the meaning of functions defined with an argument as the function input. After engaging with the task, these students demonstrated larger learning gains than students in previous studies on the same questions that involved evaluating a function for an argument expression.

Paper

6:30 – 9:00 pm *Grand Ballroom Salons 2-4*	**DINNER AND PLENARY** *Charles Henderson*

8:35 – 9:05 am

SESSION 7 – CONTRIBUTED REPORTS

Grand Ballroom Salon 5

Variation in implementation of student-centered instructional materials in undergraduate mathematics education

Christine Andrews-Larson and Valerie Kasper

Pedagogical reforms in undergraduate STEM courses are garnering increasing attention in the literature and from national organizations in disciplines such as mathematics, physics, and chemistry. While there is significant evidence to support the effectiveness of classroom-based pedagogical reforms, the way in which these reforms are taken up by those not involved in their development varies widely. This comparative case study seeks to better understand the ways in which student-centered instructional reforms in undergraduate mathematics are implemented. Data is taken from videotaped instruction of three participating instructors at three different institutions as they work to implement a student-centered instructional unit focused on supporting students' understanding of span and linear independence in undergraduate linear algebra. This analysis examines the way in which these instructors structured class time when implementing the unit, and the nature of opportunities for students to explain their thinking in the context of whole class discussions.

Paper

Marquis Ballroom A

Balancing formal, symbolic an embodied world thinking in first year calculus lectures

Sepideh Stewart, Clarissa Thompson, Keri Kornelson, Lucy Lifschitz and Noel Brady

In this paper we present a mathematics professor's thought processes while teaching Calculus I, as shared through her teaching diaries and later discussed in weekly meetings with a team of two other mathematicians, a mathematics educator, and a cognitive psychologist over the period of a semester. We examine the way she balanced formal and symbolic thinking while encouraging embodied thinking throughout her lectures. Moreover, we will discuss some data obtained from students through interviews, a questionnaire, and end-of-semester course evaluations.

Paper

*Marquis
Ballroom B*

Students' understanding of concavity and inflection points: Graphical, symbolic, verbal, and physical representations

Michael Gundlach and Steven Jones

Little research has been conducted into student understanding of concavity and inflection points. Much of what we know comes incidentally from studies looking at the calculus activity of sketching the graphs of functions. However, since concavity and inflection points can potentially be very useful in conveying information in problems in mathematics, science, engineering, and technology, it seems important to study how students understand these two concepts in and of themselves. This study attempts to provide insight into this area by providing a basic framework for what it might mean to understand concavity and inflection points and examining how students think about and reason with these two concepts.

Paper

*Marquis
Ballroom C*

An analysis of students' difficulties with the opening stage of proof construction

Tetsuya Yamamoto

This paper explores sources of students' difficulties with proving and presents pedagogical suggestions to help students with proving. This study analyzes students' difficulties in light of the structure of proof construction (a comprehensive view of proof construction, which can encompass the aspects, factors, patterns, and features involved in cognitive processes in proof construction regardless of mathematical subjects). The findings from the analysis indicate that students' difficulties might occur by multiple factors being intertwined. This study hypothesizes that the knowledge of the structure of proof construction might help students with proving.

Paper

9:15 – 9:45 am

Grand Ballroom Salon 5

Students' perceptions of the disciplinary appropriateness of their approximation strategies

Danielle Champney, David Kato, Jordan Spies and Kelsea Weber

Within the context of Taylor series expansions as approximations, we illustrate the context dependence of student reasoning about these approximations - specifically the ways in which students' notions of what is appropriate in mathematics, physics, or engineering, drive how they engage in and reflect on the solutions they produce. Using data from semi-structured interviews, we build on previous work to argue that students' epistemological framing not only plays a role in their choice of solution strategies, but also how they feel those solution strategies would be perceived within various disciplines.

Paper

City Center A

Math Teachers' Circles: Connections to teacher leadership

Diana White and Jan Yow

With the implementation of the Common Core for State Standards in Mathematics, the need for mathematics teachers to be learners and leaders in mathematics education is stronger than in the past. This article describes results from surveys of over 200 participants in Math Teachers' Circle workshops across the country that were analyzed for evidence of implications for teacher leadership. We present dat from our findings and conclude that the National Council of Supervisors of Mathematics PRIME Leadership Framework offer a promising lens with which to view Math Teachers' Circles and their impact on teacher leadership.

Paper

City Center B

Silence: A case study

Matthew Petersen

Silence has been important to many disparate traditions, notably, Zen Buddhism, and Taoism. But it has received relatively little treatment in the mathematics education literature. This paper attempts to begin a conversation on silence, its good and bad uses, and raises the question of whether silence may be an important aspect of mathematics activity. In order begin an answer to that question, it analyzes the contribution that a particular group member's silence produced for the group, leading both to a correct solution, and to a valuation of her group-mate's arguments.

Paper

Marquis Ballroom A	**Social networks among communities of calculus-teaching faculty at PhD-granting institutions**

Naneh Apkarian

Calculus is typically the first undergraduate mathematics course for science, technology, engineering, and mathematics (STEM) majors in the United States. Internationally as well as domestically, first year mathematics courses are credited with preventing students from continuing along STEM paths. A recent study of the features that characterize exemplary calculus programs from five PhD-granting institutions highlighted several common characteristics, one of which was the existence of a well-established system for coordinating Calculus I. This coordination of courses and instructors seems to engender a community of practice. This study aims to expand on this finding by leveraging social network theory to map the underlying structure of the social ties between instructors of lower-division undergraduate mathematics courses, to compare informal and formal organizational structures in each case, and to compare the communities across the five selected institutions.

Paper |
| *Marquis Ballroom B* | **The use of examples in the learning and teaching of proof writing**

Sarah Hanusch

This study investigates the ways that undergraduate students use examples in their transition to proof course, and the influence that the instructor had on the students' decisions to use examples. Data was collected from the instructor and a sample of students via observations and interviews to investigate the connections between the teaching and learning of examples in this proof writing course. The results show that the students can often state the circumstances in which an example could provide insight during proof writing, but struggle to during the implementation of the strategies.

Paper |

*Marquis
Ballroom C*

The purpose of Calculus I labs: Instructor, TA, and student beliefs and practices

Yuliya Melnikova

Currently, political and economic demand for students graduating with Science Technology Engineering and Mathematics (STEM) degrees is high, but unfortunately, a large percentage of students switch to non-STEM majors in the first year of study. Roadblock courses, such as Calculus I, can contribute to poor retention rates due to classroom environment and instructor practices. Current research suggests recitation sessions (or labs) led by teaching assistants (TAs) can positively impact student retention rates.

This study investigates the role of labs in Calculus I instruction. Through classroom observation the researcher investigated the practices of TAs and through interviews the researcher explored how beliefs about the purpose of Calculus I labs by the lead instructor, TA, and students compared to one another as well as to the practices observed. Preliminary findings on the alignment of participant views and classroom practices will be presented, and implications for increasing student retention rates will be discussed.

Paper

9:45 – 10:15 am　　**COFFEE BREAK**

10:15 – 10:45 am

*Marquis
Ballroom A*

Digging in deep: From instrumental to logical understanding in calculus

Douglas Riley and Maria Stadnik

Calculus is a foundational sequence in mathematics and many client disciplines, such as physics and engineering. For student success both in mathematics and in these client disciplines, the mathematical background provided must go beyond simple instrumental or procedural skills to a deeper level. For success in higher-level mathematics, students must delve to the level of logical understanding, being able to articulate logical connections between two mathematical concepts. In this study we analyze students' abilities to explain the connection between the limit definitions of derivative and definite integral, and their common geometric interpretations involving slope and area. We also determine whether a group exercise early in the term which reinforces the connection between the derivative and slope enhances students' written responses concerning the connection on the final exam.

Paper

*Marquis
Ballroom C*

Conditions for cognitive unity in the proving process

Kelly Bubp

Although a mathematical proof is a syntactic product, the proving process often entails other reasoning types, such as semantic or intuitive, that contribute to the evaluation of conjectures and the construction of supporting arguments. Cognitive unity and rupture refer to the possible continuity or discontinuity, respectively, between various reasoning types, argumentation and mathematical proof, and the processes of evaluating and proving conjectures. Undergraduate students struggle with mathematical proof, but it is hypothesized that cognitive unity facilitates the proving process. In this study, task-based interviews were conducted with undergraduate students who completed three prove-or-disprove tasks. The goals are to determine conditions under which students experience cognitive unity or rupture when evaluating and proving the conjectures and conditions under which cognitive unity and rupture aided or hindered the proving process. Preliminary findings suggest that various factors affect cognitive unity, cognitive unity can hinder proving, and cognitive rupture can facilitate proving.

Paper

Grand Ballroom Salon 5

Solving linear systems: Augmented matrices and the reconstruction of X

Michelle Zandieh and Christine Andrews-Larson

The origins of linear algebra lie in efforts to solve systems of linear equations and understand the nature of their solution sets. In our experience, instructors of linear algebra see the work of teaching students to solve linear systems as the more straightforward and procedural portion of the course. We speculate that solving linear systems and interpreting their solution sets in fact entails hidden and significant challenges for students that are important for their later success in linear algebra, as well as their work in related STEM courses. In this paper, we examine final exam data from 69 students in an introductory undergraduate linear algebra course at a large public university in the southwestern US. Our analysis suggests that students are largely successful in representing systems of linear equations using augmented matrices, but that interpreting the row reduced echelon form of these matrices is a common source of difficulty.

Paper

Marquis Ballroom B

Formative assessment and classroom community in calculus for life sciences

Rebecca Dibbs and Brian Christopher

Most of the attrition from STEM majors occurs between the first two semesters of calculus, and prospective life science majors are one of the groups with the highest attrition rate. One of the largest factors for students that persist in STEM major beyond the first semester of calculus was a sense of community and a perceived connection with their instructor. Since building a sense of community is one of the stated purposes of formative assessment, we investigated to what extent formative assessments could help build a sense of community in a calculus for life science majors course. Two cases of formative assessment used in two sections of this course will be discussed. When implemented as intended, the formative assessments completed weekly by the students made a positive contribution to students' sense of classroom community and their perceived connection with their instructor.

Paper

10:55 – 11:25 am

*Grand Ballroom
Salon 5*

A discursive approach to support teachers' development of student thinking about functions

Beste Gucler and Heather Trahan-Martins

This study is based on a teaching experiment in a post-secondary classroom and examines how an instructional approach that promotes high school teachers to reflect on their own discourses help them develop their thinking about student learning about functions. The findings indicate that eliciting the teachers' discourses in the classroom and making them explicit topics of discussion helped teachers reflect on and recognize their own difficulties about functions. The discursive approach used in this work also helped teachers develop their thinking of student difficulties about functions and the strategies they can use to address those difficulties.

Paper

*Marquis
Ballroom A*

Instructional practices and student persistence after Calculus I

Lisa Manitini and Kitty Debock

Classroom teaching in multiple sections of Calculus I at a large comprehensive research university was observed and coded using the Teaching Dimensions Observation Protocol (TDOP). Within lecture-based methods, multiple teaching styles were identified ranging from low to moderate to high engagement, sometimes including desk work or group work. A sample population of students from all three engagement groups was followed for one year in order to analyze persistence rates into Calculus II and retention rates in a STEM major. No significant differences were found in the retention rates either at the University or in STEM majors across groups, with an average of 43% of STEM majors having switched out of STEM or having dropped out of the University after one year. However, the group experiencing higher engagement instruction in Calculus I was found to have significantly higher grades in Calculus I and also in Calculus II.

Paper

Marquis Ballroom B

Best practices for the inverted (flipped) classroom

Spencer Bagley

The inverted, or flipped, classroom model is attracting the attention of many researchers, practitioners, and administrators in undergraduate mathematics programs as a way to navigate the tension between coverage and engagement, and to respond to the problem of increased class sizes and decreased budgets. The literature contains many reports on successful implementations, which vary widely in content delivery and student engagement. However, a core set of commonalities shared by these successful implementations forms the nucleus of a list of best practices for flipping a class. I discuss the theoretical underpinnings of the inverted model and the best practices suggested by the literature, and examine as a case study an inverted calculus class that did not follow these emerging best practices.

Paper

Marquis Ballroom C

Seeking solid ground: A study of novices' indirect proof preferences

Stacy A. Brown

The aims of this study are two-fold: (1) to investigate novices' proof preferences as indicated by novices' selection of the "most convincing" argument when engaging in proof comparison tasks involving an indirect and a direct proof; and, (2) to explore the criteria students' bring to bear on proofs as they engage in proof comparisons. Informed by the cK¢ model of conceptions proposed by Balacheff (2010), analyses indicate that directness was not a primary criterion used for the select of a proof during proof comparisons, even though this criterion was suggested by prior research. Instead, the primary criteria identified in students' rationales were familiarity and the degree of certainty in one's understanding of the given proofs, which in turn suggests that it is the subjective sense of being on solid conceptual grounds that determine students' preferences. These findings are considered in light of the cK¢ model of conceptions.

Paper

11:35 – 12:05 pm

*Grand Ballroom
Salon 5*

Variation in successful mathematics majors proving

Dov Zazkis, Keith Weber and Juan Pablo Mejia-Ramos

We examined the proof-writing behaviors of six highly successful mathematics majors on proving tasks in calculus. We found that these students approached the proof writing tasks in two different ways. Three students, who we labeled as drillers, would develop a strong understanding of the statement they were proving, choose a plan based on this understanding, develop a graphical argument for why the statement is true, and formalize this graphical argument into a proof. The other three students, who we labeled as probers, would begin trying different proving approaches immediately after reading the statement and would abandon an approach at the first sign of difficulty. Despite being inconsistent with theories of effective problem solving in the mathematics literature, the probers were highly successful in their advanced mathematics courses and on the proving tasks in this study.

Paper

*Marquis
Ballroom A*

The generalization of the function schema: The case of parametric functions

Harrison Stalvey and Draga Vidakovic

This paper reports on an investigation of fifteen second-semester calculus students' understanding of the concept of parametric function, as a special relation from a subset of R to a subset of R2. A substantial amount of research has revealed that the concept of function, in general, is very difficult for students to understand. Furthermore, several studies have investigated students' understanding of various types of functions. However, very little is known about how students reason about parametric functions. Employing APOS theory as the guiding theoretical perspective, this paper describes how students reason about parametric functions given in the form p(t)=(f(t),g(t)). One common misconception that was observed among students is addressed.

Paper

Marquis *Ballroom B*	**Linear algebra in the three worlds of mathematical thinking: The effect of permuting worlds on students' performance**

John Hannah, Sepideh Stewart and Michael Thomas

Linear algebra is a required course for STEM majors. Many undergraduate students struggle with the sudden exposure to abstraction which is almost an unavoidable feature of the course. Although research on students' conceptual understanding of linear algebra is going forward, no research has focused on how students react to order in which the concepts are taught. In this study, we use Tall's three-world model of mathematical thinking to examine students' performance in a first year linear algebra course. The study examined two sections of a linear algebra course simultaneously while the instructor changed the order in which she taught the concepts in each class. The result of this investigation so far suggests no significant difference on students' performance.

Paper

Marquis *Ballroom C*	**Promoting students' construction and activation of the multiplicatively-based summation conception of the definite integral**

Steven Jones

Prior research has shown how critical the multiplicatively-based summation conception (MBS) is for making sense of definite integral expressions in science contexts. This study attempts to accomplish two goals. First, it describes introductory lessons on integration from two veteran calculus teachers as a way to possibly explain why so few students draw on the MBS conception when making sense of definite integrals. Second, it reports the results from a design experiment intended on promoting not only the construction of the MBS conception, but its priming for activation when students see and interpret definite integrals expressions

Paper

12:05 – 1:05 pm *Grand Ballroom* *Salons 2-4*	**LUNCH**

1:05 – 1:35 pm

City Center A

Elementary mathematics pre-service teachers' consequential transitions from formal to early algebra

Charles Hohensee and Siobahn Young

Mathematics educators have long advocated for early algebra to be introduced into the elementary grades. However, little research is currently available to inform teacher preparation programs about the work of preparing undergraduate pre-service teachers for teaching early algebra. The research reported here examines the experiences of undergraduate pre-service teachers as they make consequential transitions from formal to early algebra. Preliminary results suggest that making this transition is far from trivial for undergraduates and that, to varying degrees, they face four kinds of conceptual challenges.

Paper

Grand Ballroom Salon 5

A mathematician's experience flipping a large-lecture calculus course

Erin Glover

There is a growing body of literature that suggests students do better in classes that implement active-learning strategies (e.g., group work, problem solving). Flipping classroom instruction becoming a popular innovation to support active learning in the classroom. This preliminary report highlights one instructor's experiences when implementing ambitious teaching practices in a flipped large-lecture Calculus I course.

Paper

City Center B

Implementing inquiry-oriented instructional materials: A comparison of two classrooms

Hayley Milbourne

Prior research in linear algebra education has focused on documenting and understanding the difficulties students have with specific topics. In more recent years, the research has started to shift towards developing instructional methods to address these issues. In this study, I explore the ways in which two instructors implement inquiry-oriented materials focused on span and linear (in)dependence. One of the instructors had prior experience with these materials and the other did not. Through an analysis of video recordings of these classes, I use the Inquiry-Oriented Discourse Moves framework to analyze how each instructor conducts whole-class discussion and the affordances these discussions provide their students.

Paper

Marquis Ballroom A

Partial unpacking and the use of truth tables in inquiry-based-transition-to-proofs course

Jeffrey Pair and Sarah Bleiler

During our research into an inquiry-based-transition to proofs course we observed that several students used truth tables in unique ways. Typically the students translated mathematical statements into propositional logic formalism and used the truth tables as tools in their mathematical activity. Student activity varied from using truth tables to show that a statement was a tautology, to obtain conviction in the truth of the statement, to demonstrate equivalence, or to formulate a conjecture. We hypothesize that the unique use of truth tables emerged because it was the classroom community's responsibility to socially negotiate what counted as a proof. We intend to present some of our preliminary findings, and inquire as to what further research into these cases might be worthy of pursuit.

Paper

Marquis Ballroom B

A study of connectivism as a support for research on meaning-making for mathematics

Luciane Santos, Ivanete Siple and Gabiela Lopes

This paper presents ongoing research whose goal is to study theoretical frameworks that support teaching practices carried out in courses that are part of a Bachelor's Degree in mathematics which use information and communication technology resources - such as blogs, social networks and virtual learning environments - for the teaching and learning of mathematics content that is part of the curriculum in colleges and universities. The outline presented here refers to the ongoing study in the Mathematics, Culture, Art and Technology line of research being done by the research group THEM (Spices of History in Mathematics Education) about connectivism as a theoretical approach to teaching and learning.

Paper

Marquis Ballroom C

The effects of using spreadsheets in business calculus on student attitudes

Melissa Mills

This study investigates the effects of using spreadsheets in Business Calculus. Both the computer and non-computer sections were taught using reformed curricula that focused on business applications, the use of realistic data sets, and conceptual understanding. This study compares student attitudes towards mathematics in both spreadsheet and non-spreadsheet sections as measured by pre- and post- surveys, student interviews, and examination data.

Paper

1:45 – 2:15 pm   SESSION 13 – THEORETICAL REPORTS

Grand Ballroom   **Examining individual and collective level mathematical progress**
Salon 5

Chris Rasmussen, Megan Wawro and Michelle Zandieh

A challenge in mathematics education research is to coordinate different analyses to develop a more comprehensive account of teaching and learning. We contribute to these efforts by expanding the constructs in Cobb and Yackel's (1996) interpretive framework that allow for coordinating social and individual perspectives. This expansion involves four different constructs: disciplinary practices, classroom mathematical practices, individual participation in mathematical activity, and mathematical conceptions that individuals bring to bear in their mathematical work. We illustrate these four constructs for making sense of students' mathematical progress using data from an undergraduate mathematics course in linear algebra.

Paper

Marquis   **Shape thinking and students' graphing activity**
Ballroom C

Kevin Moore and Patrick Thompson

We propose a construct called shape thinking that characterizes individuals' ways of thinking about graphs. We introduce shape thinking in two forms–static and emergent–that have materialized in our work with students and teachers over the past two decades. Static shape thinking entails thinking of a graph as an object in and of itself, and as having properties that the student associates with learned facts. Emergent shape thinking entails envisioning a graph in terms of what is made (a trace) and how it is made (covarying quantities). We provide illustrations of the two shape thinking forms using examples from data that we have gathered with secondary students, secondary teachers, and undergraduate students. We also provide future research and teaching directions with respect to students' shape thinking.

Paper

*Marquis
Ballroom A*

Value judgments attached to mathematical proofs

Eyob Demeke

In mathematics, it is a common phenomenon to find several proofs for a single result. This inevitably leads us to believe that proofs are far more than a convincing argument. Indeed, it appears that there is considerable interest in the insight that is gained from the reasoning utilized in a proof. With the existence of several proofs of the same theorem, we are then confronted with a question of value judgment, as it is not necessarily the case that one values all proofs of a given theorem equally. In this theoretical report, I attempt to provide a framework that contributes to the discussion regarding value judgments about proofs by providing a comparative language to systematically talk about judgments one may attach to a proof. I argue that proofs can be valued for reasons such as (1) comprehensibility, (2) explanatory power, (3) originality and surprises, and (4) generalizability.

[Paper](#)

*Marquis
Ballroom B*

Conceptualizing equity in undergraduate mathematics education: Lessons from K-12 research

Aditya Adiredja, Nathan Alexander and Christine Andrews-Larson

Research on equity in mathematics education has been one the primary foci among K-12 researchers. However, in research on undergraduate mathematics education, equity research has yet to maintain the presence and consistency that aligns with issues of inequity related to fairness, access and opportunity. Theories related to cognition, and socio-cultural contexts have been increasing but there has not been a shift in the focus of the research. In K-12 research, the focus has shifted from individual context to socio-cultural context and now to understanding the social and political aspect of power and privilege in mathematics education research. This report considers the socio-political perspective as conceptualized by Gutierrez (2010). In this report we consider other theories to situate the socio-political perspective as a mean to aid in the conceptualization of equity in undergraduate research.

[Paper](#)

2:25 – 2:55 pm SESSION 14 – THEORETICAL REPORTS

Grand Ballroom   **Neural correlates for action-object theories**
Salon 5

Anderson Norton

Research from an action-object perspective (e.g., APOS and reification) is well positioned to benefit from the emerging field of mathematics educational neuroscience. In this theoretical paper, we review some relevant findings from neuroscience studies and interpret them from an action-object perspective. This interpretation demonstrates a strong alignment of action-object theories and neuroscience findings, thus affirming many aspects of action-object perspectives on mathematical development. Neuroscience also serves to further elaborate and generalize reflective abstraction—the basis for action-object theories—as a mechanism for mathematical development. Specifically, we can begin to understand the changes in neural functioning associated with the objectification of action. This understanding helps explain some of the limitations teachers experience when attempting to provoke and support students' constructions of actions and objects.

Paper

Marquis   **Conceptualizing the notion of a task network**
Ballroom A

Ami Mamolo, Robyn Ruttenberg-Rozen and Walter Whiteley

We develop a theoretical model for conceptualizing the restructuring of computational / numerical tasks, usually considered advanced, with a network of spatial visual representations designed to support geometric reasoning and conceptual development. Through our restructuring of the well-known "popcorn box problem," we illustrate key developmental understandings related to optimization and rate of change, as well as the possible conceptual blends afforded by a networked spatial visual approach.

Paper

Marquis   **An extended theoretical framework for the concept of the derivative**
Ballroom B

David Roundy, Tevian Dray, Corinne A. Manogue, Joseph F. Wagner, and Eric Weber

This paper extends the theoretical framework for exploring student understanding of the concept of the derivative, which was developed by Zandieh (2000). We expand upon the concept of a physical representation for the derivative by extending Zandieh's map of the territory to provide higher resolution in regions that are of interest to those operating in a physical context. We also introduce the idea of "thick" derivatives, which are ratios of small but not infinitesimal changes, which are practically equivalent to the true derivative.

Paper

*Marquis
Ballroom C*

Frames of reference

Surani Joshua, Stacy Musgrave, Neil Hatfield and Patrick Thompson

While commonly used in math and physics, the concept of frames of reference is not described cognitively in any literature. The lack of a careful description of the mental actions involved in thinking within a frame of reference inhibits our ability to account for issues related to frames of reference in students' reasoning. In this paper we offer a theoretical model of mental actions involved in conceptualizing a frame of reference. Additionally, we also posit mental actions that are necessary for a student to reason with multiple frames of reference. This theoretical model provides an additional lens through which researchers can examine students' quantitative reasoning.

Paper

2:55 – 3:25 pm   C̲OFFEE B̲REAK

3:25 – 3:55 pm

City Center A   **"What if we put this on the floor?": Mathematical play as a mathematical practice**

J. Brooke Ernest

Mathematical play, as a mathematical practice, can be defined as the exploration of mathematical ideas in uninhibited and unconstrained ways, which could include engagement with physical devices, computer programs, imaginative acts, social interactions, and inscriptions. In this preliminary report I will discuss mathematical play and the ways in which it transpired in an undergraduate Foundation of Geometry course in which students engaged in activities to develop an understanding of physical, synthetic and analytic aspects of projective geometry. Additionally, I will discuss the ways in which mathematical play can be fostered through engagement in mathematically inspired art projects, as well as other artistic engagement.

Paper

Marquis Ballroom A   **Mathematicians' ideas when proving**

Melissa Troudt, Gulden Karakok and Michael Oehrtman

This study sought to describe the ideas professional mathematicians' find useful in moving their arguments forward while constructing mathematical proof and the context surrounding the development of these ideas. Three research mathematicians completed real analysis tasks while thinking aloud in interview and independent settings recorded through Livescribe technology. Follow-up interviews were also conducted. Data were analyzed for perceived useful ideas and coded based on Dewey's inquiry framework and Toulmin's argumentation model. Toulmin argumentation diagrams were implemented to describe the evolution of the arguments, whereas Dewey's inquiry framework helped to describe the context surrounding the development of the ideas. Preliminary findings show an active inquiry into forming a geometric understanding to develop warrants based on intuition and examples and then working to find backing that can be rendered into a formal argument.

Paper

Grand Ballroom Salon 5

Extending multiple choice format to document student thinking

Michelle Zandieh, David Plaxco, Megan Wawro, Chris Rasmussen, Hayley Milbourne, and Katherine Czeranko

The purpose of this preliminary report is to introduce a new type of assessment instrument to the mathematics education research community and to reflect with our colleagues about the possible affordances and constraints of this instrument. The questions that comprise the instrument consist of a multiple choice (MC) stem followed by a series of options from which students choose explanations (E) that support their multiple choice response. We call this style of question multiple-choice with explanation (MCE). Our decision to use MCE style questions is informed by cutting edge work in physics education research (Wilcox & Pollock, 2013) and introduces an innovative idea for assessing student thinking in mathematics. Our work is part of a larger project in linear algebra; as such, the mathematical context of the assessment instrument is linear algebra. The format of the questions, however, could be used for other subject matter as well.

Paper

Marquis Ballroom B

The influence of function and variable on students' understanding of calculus optimization problems

Renee Larue and Nicole Engelke

For this study, we aim to answer the following questions: 1) What conceptual knowledge do students need to have to be able to correctly solve optimization problems? 2) What weaknesses do students demonstrate when solving optimization problems? 3) How can we address these weaknesses and improve the teaching of optimization problems in calculus? In this paper, we discuss preliminary findings from this study, focusing on the responses of four first semester calculus students as they solve a basic optimization problem during a semi-structured interview. In particular, we observe how students' understanding of function and variable influences their understanding of optimization problems. We believe we may be able to use APOS theory (Asiala et al, 1996) as a lens for studying how students understand optimization problems and begin to explore that in this paper.

Paper

A comparison of self-inquiry in the context of mathematical problem solving

Todd Grundmeier, Dylan Retsek and Dara Stepanek

Self-inquiry is the process of posing questions to oneself while solving a problem. The self-inquiry of thirteen undergraduate mathematics students and one mathematics professor was explored. Student self-inquiry was explored via structured interviews requiring the solution of both mathematical and non-mathematical problems. The professor's self-inquiry was explored through self-reporting of questions asked in an advanced problem-solving context. Using transcripts of the student interviews, a coding scheme for questions posed was developed and all questions were coded. Data analysis of the posed questions suggests that the "good" mathematics students focus more questions on legitimizing their work and fewer questions on specification of the problem-solving task. Data analysis of the professor's self-inquiry is ongoing and will be compared and contrasted to that of the undergraduates.

Paper

4:05 – 4:35 pm

Marquis Ballroom A

Commognitive conflicts in the discourse of continuous functions

Gaya Jayakody

This paper reports on three commognitive conflicts that were identified in a larger research study conducted on university first year students' discourse on continuous functions. The study looks at continuity related aspects under a participationist, discursive lens in contrast to previous studies on continuity that have used cognitive theories that hold an acquisitionist view on learning. The study adopts Sfard's commognitive framework to analyze data. Among findings are different ways in which students use the word 'domain' and how they struggle with inconsistent 'realizations' arising from different definitions presented in text books.

Paper

Marquis Ballroom B

Cluster analysis of STEM gender differences

Ian Mouzon, Ulrike Genschel and Xuan Hien Nguyen

In this project, we form, describe, and study groups (or clusters) of students based on their academic history prior to their first semester in college. These clusters allow us to examine the effect of gender on a student's academic career decisions, such as course and major selection, while controlling for the level of preparation. We begin with an overview of standard hierarchical clustering and discuss the pitfalls of a straightforward application with our data. We then describe how to adjust the technique in order to form stable clusters. Using these clusters, we find that course selection and STEM retention are related to a student's gender, with female students more likely to leave STEM early than male students with the same level of preparation and college grades.

Paper

Grand Ballroom Salon 5

The transition from AP to college calculus: Students' perceptions of factors for success

Megan Ryals and Karen Keene

This study examines similarities and differences in the Advanced Placement and college calculus experience from the student perspective to characterize how taking AP Calculus in high school relates to student success in a college calculus course. Fourteen first-semester college students who had taken the AP Calculus exam were interviewed about their perceptions of, and experiences in the courses. The Academic Performance Determinants Model (Credé & Kuncel, 2008) was used to develop an interview protocol. Qualitative analysis of the interviews revealed four categories of themes about the students' experience: 1) Students' study approaches in the respective classes, 2) Students' self-efficacy and metacognition, 3) The Class format's effect on student success, and 4) Students' beliefs about the cognitive demand of the course. All the themes, their implications for those teaching in AP and college calculus, and the need for further research are presented.

Paper

Marquis Ballroom C

Bundles and associated intentions of expert and novice provers: The search for and use of counterexamples

Shiv Karunakaran

The argument for the importance of proving and of proof in the teaching and learning of mathematics has been repeatedly made by mathematics education researchers and by policy documents. There is also considerable research examining the existence of a gap in the proving and proof-constructing abilities of "novices" and "experts" in mathematics. However, considerably less research examines the nature of what constitutes expertise in proving mathematical statements, specifically with regard to the use of the individuals' mathematical knowledge. This study uses grounded theory methods to examine "expert" and "novice" mathematicians in the process of proving mathematical statements. The result reported here focuses on the differences in the use of and search for counterexamples by the two populations. More specifically expert provers seem to value the unsuccessful searches for counterexamples, as well as the finding of valid counterexamples.

Paper

4:45 – 5:15 pm

SESSION 17 – CONTRIBUTED REPORTS

*Marquis
Ballroom C*

Students' understanding of composition of functions using model analysis

David Miller, Nicole Engelke Infante and Solomon Adu

Model analysis is a quantitative research method used in physics education research to analyze and interpret the meaning of students' incorrect responses on a well-designed research-based multiple-choice test. We have adapted this method to study students' understanding of function composition when functions are represented graphical. Model analysis accounts for the fact that students may hold more than one idea or conception at a time, and may use different ideas and concepts in response to different situations. It is uniquely suited to study students' understanding of function composition, as students often hold multiple, sometimes conflicting misconceptions on function composition, which they may use at different times. Model analysis can capture information on self-consistency of a student's responses. We collected data from a calculus class before and after the class reviewed composition of functions. We find that model analysis offers insights not offered by traditional statistics.

Paper

*Grand Ballroom
Salon 5*

Mathematics majors' example and diagram usage when writing calculus proofs

Juan Pablo Mejia-Ramos and Keith Weber

We report on a study in which we observed 73 mathematics majors completing seven proving tasks in calculus. We use these data to empirically address several hypotheses from the undergraduate proving literature. The key findings from this study include: (a) Nearly all participants introduced diagrams and examples on multiple tasks, (b) few students relied predominantly on either semantic reasoning or syntactic reasoning, and (c) there was little correlation between one's propensity to use examples or diagrams and one's mathematical achievement, either on the proof-writing tasks or on GPA in advanced mathematics courses. Each finding is inconsistent with claims from the mathematics education literature. These inconsistencies are discussed at the end of the paper.

Paper

Marquis
Ballroom B

Students' generalizations of single-variable conceptions of the definite integral to multivariate conceptions

Steven Jones, Allison Dorko and Eric Weber

Prior research has documented several conceptualizations students have regarding the definite integral, though the conceptualizations are largely based off of single-variable integral expressions. No research to date has documented how students' understanding of integration becomes generalized for multivariate contexts. This paper describes six conceptualizations of multivariate definite integrals and how they connect to students' prior conceptions of single-variable definite integrals.

[Paper](#)

5:30 – 6:30 pm
Grand Ballroom
Salons 2-4

PLENARY SESSION
Nicole McNeil

6:30 pm

DINNER ON YOUR OWN

8:35 – 9:05 am   ## SESSION 18 – CONTRIBUTED REPORTS

Marquis
Ballroom A

Exploring practices and beliefs related to the teaching of mathematical ways of thinking and doing at university

Alon Pinto

The study presented in this paper examined lessons taught in parallel in a real analysis course by two different instructors. The two instructors implemented the same lesson-plans but decisions the instructors made prior to and during class took the lessons in substantially different directions. This paper focuses on the first lesson of one of the instructors and describes some of the ingenious ways by which he adapted the curriculum and addressed mathematical practices and ways of thinking. An analysis of the instructor's considerations, based on Schoenfeld's model for decision making, highlights relationships between the instructor's beliefs and practices and proposes explanations as to why the instructor implemented the lesson-plan and addressed the mathematics the way he did. On the basis of this analysis we discuss the nature of the process through which university instructors make sense of the written curriculum, derive and prioritize goals and form their lesson-images.

Paper

Marquis
Ballroom B

John's lemma: How one student's proof activity informed his understanding of inverse

David Plaxco

Recent discussions in the field have explored proofs' explanatory power. Such research, however, focuses on how a written proof might convey explanation. I present a conjecture that individual proof activity (the development of proofs) might, itself, have explanatory power. I then discuss one student's (John's) activity related to proving that the centralizer for a fixed element in a group (the set of elements that commute with the given element) is a subgroup and how this activity informed his understanding of inverse. During an individual interview, John developed a lemma claiming that the left- and right- inverses of an element are the same element, his proof of which contradicted his previous ways of thinking about inverse. I analyzed John's proof activity using Aberdein's (2006) extension of Toulmin's (1979) model for argumentation in order to better organize his activity, providing an example of how proof activity might itself be explanatory.

Paper

*Marquis
Ballroom C*

Teachers' meaning for average rate of change in the U.S.A. and Korea

Hyunkyoung Yoon, Cameron Byerley and Patrick W. Thompson

This study explores teachers' meanings for average rate of change in U.S.A. and Korea. We believe that teachers convey their meanings to students and teachers who have productive mathematical meanings help students build coherent meanings. We administered a diagnostic instrument to 96 U.S. teachers and 66 Korean teachers. Some of teachers' responses revealed particular problematic meanings for average rate of change that should be addressed in professional development.. Our analyses suggest that Korean teachers' meanings for average rate of change are substantially stronger than U.S. teachers' meanings.

Paper

*Grand Ballroom
Salon 5*

Studying student's preferences and performances in a cooperative mathematics classroom

Sayonita Ghosh Hajra and Natalie Hobson

In this study, we discuss our experience with cooperative learning in a mathematics content course. Twenty undergraduate students from a southern public university participated in this study. The instructional method used in the classroom was cooperative. We rely on previous research and literature to guide the implementation of cooperative learning in the class. The goal of our study is to investigate the relationship between students' preferences and performance in a cooperative learning setting. We collected data through assessments, surveys, and observations. Results show no significant difference in the comparison of students' preferences and performance. Based on this study, we provide suggestions in teaching mathematics content courses for prospective teachers in a cooperative learning setting.

Paper

9:15 – 9:45 am

City Center A   **The simple life: An exploration of student reasoning in verifying trigonometric identities**

Benjamin Wescoatt

Reasoning used by students as they verify trigonometric identities has not been investigated. Through analyzing students' spoken explanations of their thought processes in clinical interviews, this current study explores why students choose certain expressions to begin manipulating and why certain manipulations or substitutions are performed as they verify the purported identity. Preliminary findings suggest that students may use beliefs about mathematics to inform and monitor their decisions, namely, the belief that mathematical answers are results of a simplification process. Thus, students spoke of the flow of verification problems in terms of a simplifying process. This belief may be imported from a cultural belief that being simpler is better and more desired; thus, mathematical tasks could validate the imported belief and in turn strengthen the belief about mathematics as simplification. To support this possibility, this paper will share student comments. Implications for instructional practice will also be suggested.

Paper

Marquis Ballroom A   **The transfer of knowledge from groups to rings: An exploratory study**

John Paul Cook, Brian Katz and Milos Savic

Typical undergraduate course sequences in abstract algebra initiate with group theory before proceeding to ring theory. This sequencing, along with the structural similarities between groups and rings, enables many ring-theoretic concepts to be formulated in terms of results from group theory. What remains to be seen, however, is the extent to which students are able to transfer their knowledge of groups while studying topics in ring theory. Using Wagner's transfer in pieces framework, we conducted an exploratory study to investigate how students in an inquiry-oriented classroom capitalized on their knowledge of groups to make sense of rings. Preliminary results indicate both instances of obvious transfer (e.g. subgroup to subring) and also more creative approaches that might lend insight into how students think about ring structure (e.g. characterizing field-like structures as 'abelian groupings').

Paper

*Marquis
Ballroom C*

Undergraduate students' understandings of functions and key calculus concepts

Caroline Hagen

Functions are fundamental objects of study in mathematics, and research has shown that strong understandings of functions support many kinds of mathematics learning. In this paper, we explore interactions between students' understandings of functions and their learning of the key ideas of introductory calculus, such as limits and rates of change. We examine the variety of ideas that beginning calculus students' have about functions and investigating ways in which these ideas about functions interact with their learning in beginning calculus courses.

Paper

*Grand Ballroom
Salon 5*

Gains from the incorporation of an approximation framework into calculus instruction

Jason Martin and Michael Oehrtman

We report on a research-based effort to make calculus conceptually accessible to more students while simultaneously increasing the coherence, rigor, and applicability of the content learned in the courses. Recent studies have indicated that an approximation and error analysis approach to curriculum and instructional design can support a productive and coherent conceptual foundation for students' reasoning about concepts defined in terms of limit. In this study we explore the affects of such an approach to curriculum design systematically implemented in the form of 30 labs spread throughout the first two semesters of calculus. Data taken from pre-tests at the beginning of Calculus 1 and posttests near the end of Calculus 2 indicate conceptual gains above the gains previously observed from students taught without approximation curriculum.

Paper

City Center B

Studying the understanding process of derivative based on representations used by students

Sarah Dufour

The research presented in this paper aimed to construct models of understanding processes of students learning the derivative concepts. The models are constructed following Duval's theoretical framework and Hitt's ideas on representations. A teaching experiment was designed to observe students in action while they were participating to teaching episodes on the introduction of the derivative. Preliminary results for one participant are presented.

Paper

Marquis
Ballroom B

The effectiveness of clickers in large-enrollment calculus

Xuan Hien Nguyen, Heather Bolles, Adrian Jenkins and Elgin Johnston

We report the results of a two-year study of the effect of clickers in the large-lecture format through a variety of metrics. These metrics include specific quiz scores (including both conceptual and application questions), as well as pass rates (both for the general student body as well as for males and females specifically). We include statistical basis for our findings.

Paper

9:45 – 10:15 am   COFFEE BREAK

10:15 – 10:45 am

Marquis
Ballroom A

Bidirectionality and covariational reasoning

Kevin Moore and Teo Paoletti

Students' thinking about quantities that vary in tandem remains an important area of mathematics education research due to its implications for student success in mathematics. In this paper, we expand on a way of thinking about covarying quantities, called bidirectional reasoning, in ways not detailed in prior research. A student thinking bidirectionally understands two quantities varying so that the conceived relationship does not have an inherent dependency; the student understands and anticipates that quantities exist and covary simultaneously. After describing bidirectional reasoning and connecting to informing theories, we draw from our work with undergraduate students to illustrate a student reasoning bidirectionally. Because this paper serves as an introduction to bidirectional reasoning and relationships, we close with potential implications of students' bidirectional reasoning, and we hypothesize productive lines of future research.

Paper

Marquis
Ballroom B

Integrated mathematics and science knowledge for teaching framework

Shawn Firouzian and Natasha Speer

Previous studies have indicated that effective teaching relies on teachers' knowledge of both student thinking and subject content. Very little is known about the integration (combination) of teacher's mathematical knowledge and science knowledge for teaching important topics like applied derivative problems. Using Ball and colleagues' framework for Mathematical Knowledge for Teaching (MKT), data were analyzed from interviews of eight calculus Graduate Teaching Assistants (GTAs) to examine the kind of knowledge used when talking about teaching applied derivative problems. Findings suggest that some of the domains of the existing MKT framework describe the kinds of knowledge GTAs draw on. However, not all elements of knowledge these GTAs used when discussing applied problems fit the MKT framework. Modifications to the framework are proposed to describe teachers' Integrated Mathematics and Science Knowledge for Teaching.

Paper

Grand Ballroom
Salon 5

A theoretical perspective for proof construction

John Selden and Annie Selden

This theoretical paper suggests a perspective for understanding undergraduate proof construction based on the ideas of conceptual and procedural knowledge, explicit and implicit learning, behavioral schemas, automaticity, working memory, consciousness, and System 1 and System 2 cognition. In particular, we will discuss proving actions, such as the construction of proof frameworks that could be automated, thereby reducing the burden on working memory and enabling university students to devote more resources to the truly hard parts of proofs.

[Paper](#)

Marquis
Ballroom C

Adding explanatory power to descriptive power: Combining Zandieh's derivative framework with analogical reasoning

Kevin Watson and Steven Jones

The derivative is an important foundational concept in calculus that has applications in many fields of study. Existing frameworks for student understanding of the derivative are largely descriptive in nature, and there is little by way of theoretical frameworks that can explain or predict student difficulties in working with the derivative concept. In this paper we combine Zandieh's framework for understanding the derivative with "analogical reasoning" from psychology into the "merged derivative-analog framework." This framework allows us to take the useful descriptive capabilities of Zandieh's framework and add a layer of explanatory power for student difficulties in applying the derivative to novel situations.

[Paper](#)

10:55 – 11:25 am

*Grand Ballroom
Salon 5*

Mathematicians' views of mathematical practice

Elise Lockwood and Eric Weber

Learning a mathematical practice, such as problem solving, is different from learning mathematical content. Realizing that mathematical practices are a fundamental aspect of engaging in mathematical activity, we seek to better understand the nature of mathematical practices, as well as how they are perceived by those who teach them. In this paper, we explore these issues with university mathematicians. In particular, we focus on explaining how mathematicians think about, learn, and teach mathematical practices. We consider mathematicians' interpretations of various mathematical practices and consider how those interpretations may influence their goals for instruction perspectives on student thinking. Specifically we seek to know how mathematicians understand, think about, and practically address the teaching and learning of mathematical practice.

Paper

*Marquis
Ballroom A*

How Might Students Come to See First Order Differential Equations as Functions of Two Variables

George Kuster and Morgan Dominy

Using Tall and Vinner's notion of Concept Image (1981) we analyzed the concepts students used while working with the differential equation P'=3P and the connections between these concepts. We identified five interconnected notions associated with the term P' in the context of differential equations that played an integral role in the students' reasoning while attempting to solve interview tasks. In this study we report on findings concerning a pair of students' coming to seeing a differential equation as a function of two variables by reasoning with their notions of slope and rate. Specifically we address the students' transition from seeing a differential equation as an algorithm for verifying a function is a solution to a differential equation to seeing the differential equation as a relationship between a function's rate of change and evaluated value.

Paper

Marquis Ballroom B	**Investigating the effectiveness of an instructional video game for calculus: Mission Prime** *Keri Kornelson, Yu-Hao Lee, Sepideh Stewart, Scott Wilson, Norah Dunbar, William Thompson, Ryan Ralston, Milos Savic and Emily Lennox*

Marquis Ballroom B

Investigating the effectiveness of an instructional video game for calculus: Mission Prime

Keri Kornelson, Yu-Hao Lee, Sepideh Stewart, Scott Wilson, Norah Dunbar, William Thompson, Ryan Ralston, Milos Savic and Emily Lennox

Video games are becoming influential in mathematics education. There have been calls from mathematicians for video games to be developed for the purpose of mathematics education (Devlin, 2011). Although there have been many instructional video games developed and researched for K-12 education (e.g., Riconscente, 2013), little is known about instructional video games in undergraduate mathematics education. In our project, we created a video game named Mission Prime for the concept of optimization in calculus and measured the effectiveness of playing the game versus doing practice problems or no treatment at all. We conclude that there was a significant difference in the conceptual understanding of optimization between the game group and the other two, but there was no significant difference in the calculation skill between the three groups.

Paper

Marquis Ballroom C

Proof expectations of students: The effects on proof validation

Ashley L. Suominen, Hyejin Park and Annamarie Conner

We examine how fifteen prospective secondary mathematics teachers read and reflected on five different arguments purported to be proofs that the sum of the first n odd natural numbers is n^2 in an interview setting. We elaborate on how our participants validated these arguments and discuss participants' stated expectations for middle student work. Our participants had varying evaluation criteria when validating arguments, but largely focused on proof form or appearance. The majority of our participants only expected one of the arguments, which was the argument they least accepted as a proof, to be given by middle school student. In general, our participants were less likely to expect an argument from middle school students if they accepted it as a proof. Further research is needed to examine which aspects of arguments influence teachers' expectations of student performance.

Paper

11:35 – 12:05 pm

Grand Ballroom Salon 5

Instantiation practices during conjecturing activity: Implications from the use of technology

Jason Belnap and Amy Parrott

Proof is a complex mathematical activity which students struggle with as they transition from K-14 to abstract mathematics. This transition may be eased by developing mathematical practices during more accessible mathematical activities, such as conjecturing. Recent studies in both proof and conjecturing suggest that instantiation practices, practices surrounding the generation and selection of examples during mathematical inquiry, are key to success in both activities. In our own study, most participants utilized GeoGebra, a dynamic geometry software, to facilitate their investigations, but doing so did not appear to advance their instantiation practices. In this presentation, we detail the participants' use of the GeoGebra and raise implications, questions, and cautions for teaching and research.

[Paper](#)

Marquis Ballroom A

An analysis of students' difficulties with proving in light of the structure of proof construction

Tetsuya Yamamoto

This paper explores sources of students' difficulties with proving and presents pedagogical suggestions to help students with proving. This study analyzes students' difficulties in light of the structure of proof construction (a comprehensive view of proof construction, which can encompass the aspects, factors, patterns, and features involved in cognitive processes in proof construction regardless of mathematical subjects). The findings from the analysis indicate that students' difficulties might occur by multiple factors being intertwined. This study hypothesizes that the knowledge of the structure of proof construction might help students with proving.

[Paper](#)

Marquis Ballroom C	**Examining proficiency with operations on irrational numbers**

Sarah Hanusch and Sonalee Bhattacharyya

Fluency with our number system is a critical part of mathematics. Understanding how rational and irrational numbers work and fit in to the number system as a whole is at the foundation of a good understanding of mathematics (Fischbein, Jahiam, & Cohen, 1995). In this study, we present developmental mathematics students with a task which tests understanding of the closure of irrational numbers under addition and multiplication. We analyze the data with the strands of proficiency framework from Adding It Up (Kilpatrick, Swafford, & Findell, 2001), searching for evidence of each strand. The results indicate that no individual strand is particularly strong or weak among all of the students, yet small example spaces of irrational numbers may be to blame for many errors from the students. We conclude with implications for the classroom.

Paper

Marquis Ballroom B

Painter's paradox: Epistemological and didactical obstacle

Chanakya Wijeratne and Rina Zazkis

In mathematics education research paradoxes of infinity have been used in the investigation of students' conceptions of infinity. We analyze one such paradox – the Painter's Paradox – and examine the struggles of a group of Calculus students in an attempt to resolve it. This study shows that contextual considerations hinder students' ability to resolve the paradox mathematically. We suggest that the conventional approach to introducing area and volume concepts in Calculus presents a didactical obstacle. A possible alternative is considered.

Paper

12:05 – 1:50 pm
Grand Ballroom Salons 2-4

LUNCH & PANEL DISCUSSION

1:50 – 2:25 pm   **SESSION 23 – CONTRIBUTED REPORTS**

Marquis   **Modeling outcomes in combinatorial problem solving: The case of**
Ballroom A   **combinations**

Elise Lockwood, Craig A. Swinyard and John S. Caughman

In an effort to understand ways to help students solve counting problems successfully, we conducted a paired teaching experiment in which two students reinvented four counting formulas by generalizing their work from an initial set of basic problems. Subsequent to reinventing these four formulas, they solved all but one counting problem correctly, regularly drawing upon outcomes and displaying a set-oriented perspective. In this paper, we report on the problem that they missed, which involved combinations (the Bits problem: How many 256-bit binary strings contain exactly 75 0's?). We describe a key aspect of their activity that we refer to as combinatorial encoding of outcomes, and we use this language to analyze the student work. We discuss the importance of encoding as an informal way to articulate bijections, and we suggest avenues for future work and pedagogical implications.

Paper

Marquis   **Exploration of undergraduate students' and mathematicians'**
Ballroom C   **perspectives on creativity**

Gail Tang, Houssein El Turkey, Milos Savic and Gulden Karakok

Mathematical creativity has been an object of discussion in mathematics for some time (Borwein, Liljedahl, & Zhai, 2014). Though there are recent efforts to include creativity in K-12 education agendas (Askew, 2013) and a number of research articles in mathematics education literature about creativity focus K-12 (e.g., Silver, 1997), there is little research in undergraduate mathematics education about creativity. Our study aims to address this issue by examining what mathematicians and undergraduate students think about creativity in proving. We coded six mathematician and eight student interviews using the Creativity in Progress Rubric (CPR) on proving created by the research group. A majority of the students' responses were of the Creating Ideas category of the rubric, while mathematicians were more balanced. We claim that the other two categories (Taking Risks and Making Connections) might need further explicit discussion in classrooms if the transition from student to mathematician is desired.

Paper

Guiding reinvention of conventional tools of mathematical logic: Students' reasoning about mathematical disjunctions

Paul Dawkins and John Paul Cook

Motivated by the observation that formal logic answers questions students have not yet asked, we conducted a set of exploratory teaching experiments with undergraduate students intended to guide their reinvention of truth-functional definitions for basic logical connectives. We intend to bridge the gap between reasoning and logic by inviting students to ask and answer the questions that motivated logic as an objective science. We present categories of student strategies for assessing truth-values for mathematical disjunctions. As we expected, students' reasoning heavily reflected content-specific and pragmatic factors, inconsistent with the norms and conventions of formalized logic. Despite this, all groups reinvented the standard truth-functional definition for non-quantified disjunctions once they began reasoning about logic by attending directly to the logical connective or and comparing their interpretations across various disjunctions. Students struggled to develop generalizable tools for assessing quantified disjunctions because they explored sets of examples in context-dependent ways.

Paper

2:30 – 3:00 pm SESSION 24 – PRELIMINARY REPORTS

City Center A **Pre-service teachers' conceptual understanding of arithmetic in base-ten and bases other than ten**

Benjamin Wescoatt and Iwan Elstak

This preliminary study explores pre-service teachers' learning and understanding of arithmetic as they are confronted with numbers represented in different bases in order to add to the body of literature about teacher knowledge. Students in an initial mathematics content course for elementary teachers were interviewed as they solved problems related to representing whole numbers and related to arithmetic operations on numbers represented in various bases. Initial findings suggested that students relied on their knowledge of the decimal system in order to solve problems involving numbers represented in bases other than ten. Additionally, students appeared to not attach conceptual meaning to words they used, suggesting they relied on procedures. Thus, the students became proficient with procedures but lacked conceptual understanding.

Paper

Marquis Ballroom A **The purpose of reading a proof: A case study of Lagrange's Theorem**

Eyob Demeke and May Chaar

In typical undergraduate advanced mathematics courses, professors spend ample class time presenting proofs; however little is known with regards to what students actually gain from these experiences. In our preliminary report we will attempt to address this gap, specifically with respect to a proof of Lagrange's theorem. We used Mejia-Ramos and colleagues' (2012) model in designing a proof comprehension test and task-based interviews to shed light on (1) the extent to which undergraduates comprehend the proof and (2) what undergraduates gain or learn from reading the proof. Initial examination of our data reveals that although the participants could follow the proof line by line, they had difficulty identifying key ideas and summarizing the proof. Participants acknowledged their responsibility to fill in gaps in proofs; yet they had trouble justifying non-trivial assertions. Despite participants' superficial comprehension of the proof, we still observed that participants gained conviction and learned new definitions.

Paper

Grand Ballroom Salon 5

Calculus students' understanding of making predictions using slope and derivative

Jennifer Tyne

Studies have shown that students have difficulty with the concepts of slope and derivative, especially in the case of real-life contexts. Following up from a previous study, written surveys were used to collect data from 20 differential calculus students and pilot interviews were conducted. On the surveys, students answered questions about linear and nonlinear relationships and interpretations of slope and derivative. They also critiqued the reasoning and accuracy of a hypothetical person's predictions. In pilot interviews, students explained their thought process and reasoning, and answered follow-up questions. Preliminary results indicate that students struggle with knowing what the derivative represents and how to use it appropriately to make predictions. Overall, students preformed much better on linear problems then non-linear problems, and showed a limited understanding of the appropriateness of using derivatives to make predictions. Plans for an expanded survey and additional interviews are in place for fall 2014.

Paper

Marquis Ballroom B

Secondary mathematics teachers' perceptions of real analysis in relation to their teaching practice

Nicholas Wasserman, Matthew Villanueva, Juan Pablo Mejia-Ramos and Keith Weber

Fourteen secondary mathematics teachers were given a task-based interview in which they were presented with four mathematical tasks from secondary school mathematics that a real analysis course might prepare them to handle. We found that most participants could not answer these questions correctly. Some participants believed the answers to some of these questions could inform their teaching, but felt these topics were not included in their real analysis course. We suggest that explicitly discussing the corollaries relevant to secondary school mathematics in real analysis courses for prospective teachers might be a useful first step in helping future teachers see real analysis as relevant to their instruction.

Paper

*Marquis
Ballroom C*

Creating opportunities for students to address misconceptions: Student engagement with a task from a reform-oriented calculus curriculum

Sarah Enoch and Jennifer Noll

Research shows that students learn best when working with their peers as this creates opportunities for students to challenge one another's thinking, consequently building new knowledge. We will share analysis of three groups of calculus students engaging with an activity from a new reform-oriented calculus curriculum, which developed out of the philosophy that students working together promotes increased learning. Inspired by Gresalfi's engagement framework (2013), we developed an engagement coding scheme that differentiates between weak, procedural, and conceptual engagement. Our analysis focuses on the opportunities that emerged for students to address misconceptions. Findings show that how groups engaged with the task varied significantly and, although opportunities to address misconceptions emerged in all groups, misconceptions were most effectively addressed in the group where conceptual engagement was prevalent. We discuss the extent to which norms, learning dispositions, and the activity itself played a role in the outcomes we observed.

Paper

3:00 – 3:30 pm   **COFFEE BREAK**

3:30 – 4:00 pm

City Center A   **Marginalizing, centralizing, and homogenizing: An examination of inductive-extending generalizing among preservice secondary educators**

Duane Graysay

Policy and standards documents from recent decades emphasize the importance of attending to students' development of ways of engaging in mathematical activities such as generalizing. Therefore, we need to be able to describe proficient generalizing and ways of explaining and predicting the strategies that individuals use to generalize. However, existing research seems restricted to examining generalizing at the K-12 level on a specific type of task. This report presents tentative findings from the preliminary analysis of data collected as part of the author's dissertation research.

[Paper]

City Center B   **Impacts on learning and attitudes in an inverted introductory statistics course**

Emily Cilli-Turner

Recent studies have highlighted the positive effects on learning and retention rates that active learning brings to the classroom. A flipped classroom is a type of active learning where transmission of content occurs outside of the classroom environment and problem solving and learning activities become the focus of classroom time. This article reports on results of a study conducted in flipped and non-flipped introductory statistics classroom environments measuring student achievement in both classrooms on traditional assessments as well as measuring student attitudes toward the flipped classroom environment.

[Paper]

Marquis Ballroom A

Calculus students' understanding of logical implication and its relationship to their understanding of calculus theorems

Joshua Case

In undergraduate mathematics, deductive reasoning is an important skill in the learning of theoretical ideas. Deductive reasoning is primarily characterized by the concept of logical implication (inferring what follows from a given premise). This plays a role whenever mathematical theorems are applied, i.e. one must first check if a theorem's hypothesis is satisfied and then make a correct inference. In Calculus, students must learn how to apply theorems. However, most undergraduates have not yet received extensive training in propositional logic. How do these students comprehend the notion of logical implication and how does it relate to their understanding of theorems? Results from a pilot study indicated that students struggled with the notion of logical implication in both symbolic and Calculus contexts. However, findings were inconclusive regarding the relationship between the two areas. Background on the current literature, results of the pilot study, and further avenues of inquiry are discussed.

Paper

Marquis Ballroom B

A mathematics teacher educator's use of technology in a content course focused on covariational reasoning

Kevin Laforest

The actions of mathematics teacher educators (MTEs) are an underrepresented area of mathematics education research. Likewise, researchers have argued that a pressing area of need in mathematics education is investigating how to support students' covariational reasoning. The purpose of this proposed study is to investigate an MTE's use of technology to engender covariational reasoning in a content course designed for pre-service teachers. Through the lens of Carlson et al.'s (2002) covariation framework and related theories, I will analyze interview and observation data in order to provide insights into the ways in which an MTE conceptualizes the use of technology in his classroom as well as how he plans to implement it to engender covariational reasoning. Additionally, I will focus on the way in which the MTE implements the technology including how students in the classroom reason about quantities that vary through the instructor's use of technology.

Paper

Grand Ballroom Salon 5

Calculus students' meanings for difference

Stacy Musgrave, Neil Hatfield and Patrick Thompson

Students learn the mathematical operation of subtraction beginning in elementary school, along with key vocabulary to talk about that operation. However, the meanings that students develop for the word "difference" continue to play a role well into students' study of undergraduate mathematics. In particular, a meaning for "difference" as representing a change in a quantity is essential to understanding and communicating about foundational ideas in Calculus. In this preliminary report, we consider meanings about the word "difference" held by calculus students as revealed on a pre-test in an on-going study designed to explore Calculus students' structure sense. We further propose potential consequences for those meanings and describe methods to be used in data collection for the remainder of the Fall 2014 semester.

Paper

Marquis Ballroom C

Public versus private mathematical activity as evaluated through the lens of examples

Tim Fukawa-Connelly

Lecture has often been critiqued as obscuring the mathematical habits such as sense-making about abstract statements about mathematical concepts, the creation of conjectures about those concepts, and to the processes of proof-writing and exhibition of counter-examples. In all of these actions, a mathematician or student should be able to draw upon a rich store of examples in order to make meaningful progress and researchers have argued that it is via examples that they are best able to engage in such processes. Inquiry-based classes have, as part of their promise, making more visible the mathematical promises that lecture obscures behind the polished formalism. This preliminary report explores whether and how students in an inquiry-based abstract algebra class engage in public example-based reasoning as a means to explore public versus private mathematizing.

Paper

4:10 – 4:40 pm   SESSION 26 – CONTRIBUTED REPORTS

Grand Ballroom   **Undergraduate students' construction of existence proofs**
Salon 5

Kyeong Hah Roh and Yong Hah Lee

The purpose of this study is to explore students' activities while they construct existence proofs. We focus on three undergraduate students who completed a transition-to-proof course, and analyze their constructions of existence proofs. The results indicate that students' activities for existence proofs were associated with their interpretations of a given statement, strategic knowledge for their proofs, and proving frames recruited. In addition, we discuss how students' conceptions of proof also play a role in their construction of existence proofs.

Paper

Marquis   **Examining the pedagogical implications of a secondary teacher's**
Ballroom A   **understanding of angle measure**

Michael Tallman

This paper reports the results of a series of task-based clinical interviews I conducted to examine how a secondary mathematics teacher's understandings of angle measure afford or constrain his capacity to bring his mathematical knowledge to bear in the context of teaching. The results suggest that the teacher, David, possessed two complimentary but conceptually distinct ways of understanding angle measure that he was not consciously aware of having. As a result David was unable to strategically employ his two ways of understanding in novel problem-solving situations and was unable to leverage his understandings in the context of teaching. I also discuss the effectiveness of an instructional intervention designed to support David in becoming aware of his understandings and conclude that engaging teachers in experiences that promote reflected abstraction is one way of supporting them in transforming their mathematical knowledge into a pedagogically efficacious form.

Paper

*Marquis
Ballroom B*

The equation has particles! How calculus students construct definite integral models

Kritika Chhetri and Michael Oehrtman

This research characterizes the cognitive challenges that students encounter while constructing definite integrals to model physical quantities and relationships, how students resolve those challenges, and the resulting conceptual artifacts. We video-recorded four groups of second-semester calculus students working with definite integral models during two 50-minute labs. This paper focuses on how two groups of students that worked on the same problem reasoned about and constructed an integral representing the gravitational attraction between a co-linear thin rod and point mass. Prior research on students' understanding of definite integrals posits the multiplicative structure in a Riemann sum as an essential component in conceiving an integral. Our findings indicate that in many contexts, other symbolic forms subsume the simple product in this essential conceptual role and that they interact significantly with students' symbolic forms for the definite integral.

[Paper](#)

*Marquis
Ballroom C*

Determining what to assess: A methodology for concept domain analysis as applied to group theory

Kathleen Melhuish

With the rise of concept inventory style student assessment, focus has been placed on research-based refinement of assessment questions. Of equal importance is how one arrives at the tasks meant to reflect a given construct. This report will consider how this might be done in a research-based manner. The creation of a Group Concept Inventory (with particular attention to isomorphism) will be utilized to illustrate the approach. The methodology attempts to add rigor to vague suggestions of using textbooks, consulting experts, and referencing literature when developing assessment tasks. Various aspects of understanding concepts and attention to which concepts are essential to group (at an introductory level) will be discussed. The methodology and accompanying example will detail a Delphi process for expert consensus, a narrative and exercise textbook analysis and a thorough literature exploration.

[Paper](#)

4:50 – 5:20 pm

Grand Ballroom   **Semantic and logical negation: Students' interpretations of negative**
Salon 5   **predicates**

Paul Dawkins and John Paul Cook

During exploratory teaching experiments intended to guide students to reinvent basic truth-functional definitions for basic logical connectives, it unexpectedly emerged that undergraduate students reasoned about negation and negative properties in ways incompatible with the conventions and norms of mathematical practice. Specifically, our study participants often unpacked negative properties ("not a rectangle") in terms of positive properties ("is a parallelogram"), which we call semantic negation. In this way, students did not readily adopt the mathematical assumption that the logical negation of a predicate designates the complement of the set of examples that satisfy that predicate. Students especially understood geometric sets of objects as being partitioned by familiar categories rather than those stipulated in a given statement. Semantic negation inhibited students' systematizing activity regarding linguistic interpretation because their reasoning about disjunctions in various mathematical contexts depended intrinsically upon their understanding of particular topics so as to preclude abstractions approximating normative logical tools.

Paper

Marquis   **Students' reasoning when constructing quantitatively rich situations**
Ballroom A

Teo Paoletti

Researchers continue to emphasize the importance of examining students' reasoning when constructing situations involving numerous quantities and relationships (e.g., quantitatively rich situations). To explore student reasoning in such situations, I conducted a semester-long teaching experiment with two mathematics education undergraduate students. The teaching experiment sessions were focused on providing the students repeated opportunities to conceptualize quantitatively rich situations. In this proposal, I explore a few themes that emerged through analyses of their activity, characterizing their thinking during their construction of such situations. For instance, the order in which a student coordinated two quantities (e.g., coordinating a change in quantity A then a corresponding change in quantity B versus coordinating a change in quantity B then a corresponding change in quantity A) emerged as critical to their images and their representations of the situation. This and other findings provide important insights into ways students' reason quantitatively and covariationally.

Paper

An analysis of proof-based final exams

Mindy Capaldi

Evaluating final exams can give insight into what aspects of a course instructors value the most. This study examines whether final exams in proof-based courses accurately reflect instructors' perceptions of their exams. It also categorizes the questions on the exams in terms of type, format, and cognition level. The level of cognition was further distinguished by imitative reasoning and creative reasoning. Results indicate that instructors of proof-based classes are relatively aware of the content of their exams. The largest discrepancy between instructor perception and reality concerned the number of application problems. Analysis also showed that about a fifth of the questions and over half of the points are proof-related. The exams were almost entirely short or broad answer and required high cognition levels when proofs were assumed to require creativity.

Paper

5:20 – 6:15 pm

Grand Foyer

Unifying concepts in the introductory linear algebra course

Spencer Payton

The introductory linear algebra course provides many unique challenges to undergraduate students. With so many new concepts and definitions, students often struggle to see the inherent connections between these concepts. In this action research study, I attempted to discover alternative ways of presenting these connections to undergraduates. I observed and collected data from several introductory linear algebra classes, including my own. Data was collected from student responses to worksheets, midterm examinations, and interviews. In my presentation of the material to my class, I attempted to illustrate connections through solution sets of matrix equations. This presentation led to several students displaying what I describe as a linear systematic concept image. These students seemed more able to see and make connections between linear algebraic concepts.

Paper

Analyzing data from student learning

Bernard Ricca and Kris Green

A full examination of learning or developing systems requires data analysis approaches beyond the commonplace pre-/post-testing. Drawing on graph theory, three particular approaches to the analysis of data – based on adjacency matrices, affiliation networks, and edit distances – can provide additional insight into data. Data analysis methods based on adjacency matrices demonstrate that learning is not unidimensional, and that learning progressions do not always progress monotonically toward desired understandings and also provide insight into the connection between instruction and student learning. The use of affiliation networks provides insight into how students' prior knowledge relates to topics being studied. Careful use of edit distances indicates a likely overestimate of effect sizes in many studies, and also provides evidence that concepts are often created in an ad hoc manner. All of these have implications for curriculum and instruction, and indicate some directions for further inquiry.

Paper

An exploration of students' conceptions of rational functions while working in a CAS-enriched dynamic environment

Derek Williams

Studying families of functions is important for developing an understanding of the concept of function. This study utilized a computer algebra system (CAS) learning environment as it explored how students conceptualize rational functions. Using APOS theory as a theoretical framework, student artifacts from video, audio, and screen recordings were coded as either: action, process, or object conceptions of rational functions. The results of this study describe how students portray different conceptions of rational functions. This list is not exhaustive, but provides a starting point for identifying how students perceive of rational functions in a technological environment. Developing an understanding of how students conceptualize rational functions could lead to useful implications for teaching.

[Paper](#)

Creating online videos to help students to overcome exam anxiety in statistics class

Anna Titova

In this poster I would like to share my ideas on how to help students overcome math anxiety, exam anxiety in particular. Math anxiety is reported to be a major learning obstacle for many students at various levels of learning mathematics; students worry about outcomes of in-class assessments, especially exams. Instructors typically place a lot of weight on exams, so students fear that if they do poorly their grade will go down. This is known as exam anxiety; not necessarily math test anxiety, but putting the two together would certainly multiply students' stress. In this poster I would like to illustrate how videos can be utilized to ease math test anxiety and help improve students' overall performance in math classrooms.

[Paper](#)

Students' generalizations from single variable function to multi variable function in the context of limit

Sarah Kerrigan, Erin Glover, Eric Weber and Allison Dorko

Studies indicate that students struggle with generalizing across mathematical contexts, yet little research on generalization has been conducted in post secondary settings. This poster presentation reports on students' generalization in the context of limit because understanding of limit is an essential part of what it means to understand calculus. Interviews were conducted in order to characterize students' generalizations as they made sense of limits in higher dimensions.

[Paper](#)

Multiple representations of the group concept

Annie Bergman, Kate Melhuish and Dana Kirin

This poster will explore the various representations of groups found within introductory abstract algebra textbooks. Representations play an essential role in students understanding of mathematics (Goldin, 2002). Textbooks provide one source for analyzing the intended curriculum and what representations students may have access to.

[Paper](#)

The role of examples in understanding quotient groups

Carolyn James

This poster investigates two students' use of examples in the transition to advanced mathematical thinking in the context of a teaching experiment focusing on quotient groups. Based upon the theoretical perspective of Edwards, Dubinsky, and McDonald (2008), this poster extends the construct of imperfect models to include instances of generic examples. This study found that students leveraged examples to construct conjectures, form justifications, clarify misconceptions, and provide analogies for further reasoning. However, attending to the specific features of particular examples also led to over-generalizations and invalid reasoning.

[Paper](#)

Psychometric analysis of the Calculus Concept Inventory

Matt Thomas, Jim Gleason, Spencer Bagley, Lisa Rice, Nathan Clements and Diana White

Concept inventories have become an increasingly popular way to measure conceptual understanding in STEM disciplines. Interest in the Calculus Concept Inventory (CCI) has grown recently, though the descriptions of the validation and analysis have been less clear than in other concept inventories. Moreover there is a lack of peer-reviewed literature on its development and psychometric analysis. A valid and reliable instrument for measuring conceptual understanding of differential calculus is essential in the work of determining evidence-based approaches for the teaching and learning of differential calculus. We present a psychometric analysis of CCI data that was collected from over 1500 students at four institutions. The findings raises questions about the instrument's ability to measure conceptual, rather than simple procedural, knowledge based on increased notational and vocabulary knowledge.

[Paper](#)

Student understanding of solution

Rebecca Walker

Student understanding of solution is central to success in much of mathematics, from basic algebra through linear algebra and differential equations. This research explores college algebra students' understanding of solution. In particular, it explores student definitions of what it means for a number to be a solution to an equation and whether students can determine if a number is a solution to an equation. Results show that most of the students could determine if a given number is a solution to an equation but that fewer than half of the students could write a reasonable definition of solution. Categories of student responses are identified along with possible reasons for the misconceptions. These results have implications for teaching all levels of mathematics.

[Paper](#)

Beyond good teaching: The benefits and challenges of implementing ambitious teaching

Kathleen Melhuish, Erin Glover, Sean Larsen, and Annie Bergman

Lampert et al. (2010) define ambitious teaching as teaching designed to achieve the ambitious goals of conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive dispositions. They argue that this kind of teaching necessarily involves actively engaging students. Yet, lecture continues to dominate introductory college calculus courses throughout the country. In this poster, we draw on data from an national project (Characteristics of Successful Calculus Programs) to explore both national trends regarding ambitious teaching practices in calculus and present two case studies that give some important insights into how department-wide ambitious teaching can be instituted and sustained.

[Paper](#)

Can mathematics be a STEM pump?

William Bond and John Mayer

It is widely recognized that sustaining the economic leadership of the United States over the next two decades will require a robust supply of bachelor's degree graduates in the STEM disciplines. Certain courses within the first two years in mathematics and the sciences are rate-limiting for students that nevertheless have a realistic chance of success in a major in science, technology, engineering, or mathematics (STEM), but "bog down" at a particular point in their trajectory of courses. Identification of such points, and application of resources to improve student success at those points, has the potential to dramatically improve retention in STEM. This poster reports on a Pilot Study (Bond, 2013) of two points in mathematics student trajectories (first university mathematics course taken and calculus 1), and an outline of a planned study of STEM student trajectories, more generally, in the sciences. engineering, and mathematics.

[Paper](#)

Some preliminary results on the influence of dynamic visualizations on undergraduate calculus learning

Julie M. S. Sutton

To produce more STEM graduates in the U.S., improving student success in calculus is crucial; previous research suggests that students with a proclivity to visualize when solving mathematics tasks are not the "star" students in mathematics classrooms. A study of undergraduate curriculum also found that common calculus tasks reinforce procedural understanding. Since incorporating dynamic visualization (DV) provides a possible tool for increasing understanding, we investigate the role of DV in calculus learning at the university level. We examine student understanding of derivative as a rate of change by comparing student experiences when exploring with DV software or engaging with static tasks in individual interviews collected in four episodes over one semester on four students identified as visualizers and five as non-visualizers. Comparisons reveal the emergence of cognitive conflict and its resolution for students encountering the DVs but this resolution is not evident for those only engaged in static work.

[Paper](#)

Investigating backward transfer effects in calculus students

Siobahn Young

One of the most important concepts in calculus is the derivative. Unfortunately, many studies have shown that students have trouble understanding derivatives, possibly due to deficiencies in knowledge about concepts learned prior to calculus. However, few studies have investigated how learning about other calculus topics affects students' understandings of the derivative. In this study, qualitative methods were used to investigate the influence, or backward transfer effect, that learning about integration has on students' prior understandings of derivatives. Semi-structured task-based interviews were conducted with four high school students before and after students received instruction on integration. Interview tasks involved finding derivatives and antiderivatives algebraically and graphically. Results from this study may show that learning about integration is another potential reason for why students have trouble understanding derivatives. Alternately, results may show that it is possible for teachers to help reinforce students' understandings of derivatives through instruction on other calculus topics.

[Paper](#)

A study of mathematical behaviors

Nadia Hardy

In this poster presentation we bring together different characterizations of mathematical thinking, doing and behaving that researchers have brought forward over the last three decades, to compose, in a way of speaking, a collage of what we came to call mathematical behaviors. In previous work, we have combined some of these characterizations to identify opportunities to engage in mathematical behaviors that students may encounter in undergraduate courses, and to design and analyze tasks that may foster the development of such behaviors. The poster format allows us to play with a pictorial representation that reveals the relations and complementarities between the 'images' of the collage. We hope to discuss (a) these relations and complementarities, (b) strategies to foster the development of mathematical behaviors in undergraduate mathematics students, and (c) methods that may allow us, as researchers or as teachers, to compose accounts of students' mathematical behaviors.

[Paper](#)

Effects of engaging students in the practices of mathematics on their concept of mathematics

Duane Graysay, Shahrzad Jamshidi, and Monica Smith Karukaran

Two important parts of mathematical proficiency are the individual's understanding of mathematics and his or her self-efficacy beliefs with respect to mathematics. Understanding the nature of mathematics should include recognizing mathematics as a field of inquiry. However, it is not clear how changes in a student's understanding of the nature of mathematics might affect the ways that the student perceives his or her mathematical abilities. We designed a 5-week course around inquiry projects in an attempt to promote a more robust understanding of the nature of mathematics. Using surveys and interviews, we gathered information about students' perceptions of mathematics and about their own mathematical abilities. This preliminary study suggests that engaging in an inquiry-based course experience helped students to recognize the roles of collaboration and communication in mathematics, but may also have led them to perceive mathematics as inherently more difficult and themselves as less able to communicate mathematically.

Paper

6:30 – 9:00 pm	**DINNER AND PLENARY**
Grand Ballroom Salons 2-4	*Matthew Inglis*